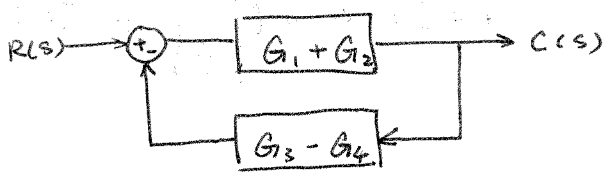
MAE 170 Homework 2 Solutions.

3.1



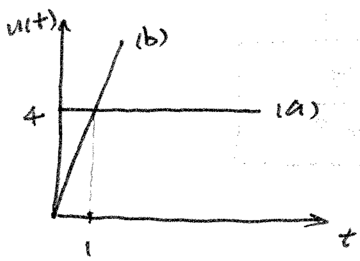
$$\frac{C}{R} = \frac{G_1 + G_2}{1 + (G_1 + G_2)(G_3 - G_4)}$$

3.4

$$\frac{U(s)}{E(s)} = K_p = 4$$

(a) $u(t) = 4 \cdot 1(t)$

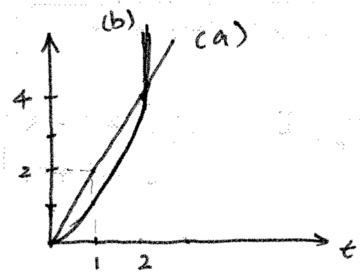
(b) $u(t) = 4t$



$$\frac{U(s)}{E(s)} = \frac{K_i}{s} = \frac{2}{s}$$

(a) $u(t) = 2t$

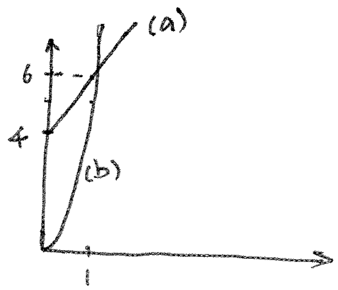
(b) $u(t) = t^2$



$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right) = 4\left(1 + \frac{1}{2s}\right)$$

(a) $u(t) = \mathcal{L}^{-1}\left(\frac{4}{s} + \frac{4}{2}\frac{1}{s^2}\right) = 4 \cdot 1(t) + 2t$

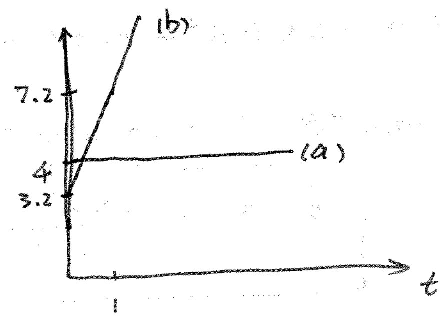
(b) $u(t) = \mathcal{L}^{-1}\left(\frac{4}{s^2} + 2\frac{1}{s^3}\right) = 4t + t^2$



$$\frac{U(s)}{E(s)} = 4(1 + 0.8s)$$

(a) $u(t) = \mathcal{L}^{-1}\left[4\frac{1}{s} + 3.2\right] = 4 \cdot 1(t) + 3.2\,\delta(t)$

(b) $u(t) = \mathcal{L}^{-1}\left[\frac{4}{s^2} + \frac{3.2}{s}\right] = 4t + 3.2 \cdot 1(t)$



3.9 $\quad y''' + 3y'' + 2y' = u$

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} + [0]\, u$$

3.11 $\quad A = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \qquad D = 0$

$$G(s) = C(sI - A)^{-1}B + D = C \frac{1}{\det(sI-A)} \left[\operatorname{adj}(sI-A) \right] B$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{(s+5)(s+1) + 3} \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \frac{1}{s^2+6s+8} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 2s-3 \\ 5s+31 \end{bmatrix} = \boxed{\frac{12s + 59}{s^2 + 6s + 8}}$$

using matlab

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[num, den] = ss2tf(A, B, C, D);
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3.16 $\quad$ assuming small θ, $\qquad T = J\ddot{\theta}$

$\Rightarrow J = ml^2 \qquad \sin\theta = \theta$

$$T = -2k\theta a^2 - mgl\sin\theta = J\ddot{\theta}$$

$$ml^2\ddot{\theta} = -2ka^2\theta - mgl\theta$$

$$\boxed{\ddot{\theta} = -\frac{2ka^2}{ml^2}\theta - \frac{g}{l}\theta}$$

3.18

Differential Equation of motion for the system

$$m_1 \ddot{x}_1 = -k_1 x_1 - b_1 \dot{x}_1 - k_3 (x_1 - x_2) + u$$

$$m_2 \ddot{x}_2 = -k_2 x_2 - b_2 \dot{x}_2 - k_3 (x_2 - x_1)$$

Hence

$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k_1 x_1 + k_3 x_1 = k_3 x_2 + u$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_2 x_2 + k_3 x_2 = k_3 x_1$$

perform Laplace transform

$$(m_1 s^2 + b_1 s + k_1 + k_3)\, X_1(s) = k_3 X_2(s) + U(s)$$

$$(m_2 s^2 + b_2 s + k_2 + k_3)\, X_2(s) = k_3 X_1(s)$$

$$(m_1 s^2 + b_1 s + k_1 + k_3)\, X_1(s) = \frac{k_3\, k_3\, X_1(s)}{m_2 s^2 + b_2 s + k_2 + k_3} + U(s)$$

$$\Rightarrow \boxed{\frac{X_1(s)}{U(s)} = \frac{m_2 s^2 + b_2 s + k_2 + k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}}$$

since $X_2(s) = \dfrac{k_3 X_1(s)}{m_2 s^2 + b_2 s + k_2 + k_3}$

$$\Rightarrow \boxed{\frac{X_2(s)}{U(s)} = \frac{X_2(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = \frac{k_3}{(m_1 s^2 + b_1 s + k_1 + k_3)(m_2 s^2 + b_2 s + k_2 + k_3) - k_3^2}}$$

3.1 (1 pt)
3.4 (4 pts)
3.9 (2 pts)
3.11 (2 pts)
3.16 (2 pts)
3.18 (2 pts)

Total $\underline{\underline{13}}$ pts