0.1 Problem B 2-1

From Modern Control Engineering, 4th edition by Ogata

Question

1. Find Laplace transform for

$$\begin{aligned} f(t) &= 0 && t < 0 \\ f(t) &= e^{-0.4t}\cos 12t && t \geq 0 \end{aligned}$$

2. Find Laplace transform for

$$\begin{aligned} f(t) &= 0 && t < 0 \\ f(t) &= \sin\left(4t + \tfrac{\pi}{3}\right) && t \geq 0 \end{aligned}$$

Solution

0.1.1 Part a

This is of the form $e^{-at}f(t)$, hence use the property of Laplace transform

$$\mathscr{L}\left(e^{-at}f(t)\right) = F(s+a) \tag{1}$$

Where $F(s)$ is Laplace transform of $f(t)$. But $\mathscr{L}(\cos\omega t) = \frac{s}{s^2+\omega^2}$, therefore

$$F(s) = \mathscr{L}(\cos 12t) = \frac{s}{s^2+144}$$

Hence (1) becomes

$$\begin{aligned} \mathscr{L}\left(e^{-at}f(t)\right) &= \mathscr{L}\left(e^{-0.4t}\cos 12t\right) \\ &= F(s+a) \\ &= \frac{(s+0.4)}{(s+0.4)^2+144} \end{aligned}$$

0.1.2 Part b

I can not solve $\mathscr{L}\left(\sin\left(4t+\frac{\pi}{3}\right)\right)$ by using the property that

$$\mathscr{L}\left(f(t-a)\right) = e^{-as}F(s)$$

Because here delay $\frac{\pi}{3} > 0$ where the above property is valid for $a < 0$. Instead, writing

$$\begin{aligned} \sin(\omega t+\theta) &= \sin(\omega t)\cos\theta + \cos(\omega t)\sin\theta \\ \sin\left(4t+\frac{\pi}{3}\right) &= \sin(4t)\cos\frac{\pi}{3} + \cos(4t)\sin\frac{\pi}{3} \\ \mathscr{L}\left(\sin\left(4t+\frac{\pi}{3}\right)\right) &= \cos\frac{\pi}{3}\mathscr{L}(\sin 4t) + \sin\frac{\pi}{3}\mathscr{L}(\cos 4t) \end{aligned} \tag{2}$$

But $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \sqrt{3}$ and $\mathscr{L}(\sin\omega t) = \frac{\omega}{s^2+\omega^2} \implies \mathscr{L}(\sin 4t) = \frac{4}{s^2+16}$ and $\mathscr{L}(\cos\omega t) = \frac{s}{s^2+\omega^2} \implies \mathscr{L}(\cos 4t) = \frac{s}{s^2+16}$. Hence substituting into eq (2) gives

$$\begin{aligned} \mathscr{L}\left(\sin\left(4t+\frac{\pi}{3}\right)\right) &= \frac{1}{2}\frac{4}{s^2+16} + \sqrt{3}\frac{s}{s^2+16} \\ &= \frac{1}{2}\left(\frac{4+\sqrt{3}s}{s^2+16}\right) \end{aligned}$$