

0.1 Problem B 2-1

From Modern Control Engineering, 4th edition by Ogata

Question

1. Find Laplace transform for

$$\begin{aligned} f(t) &= 0 & t < 0 \\ f(t) &= e^{-0.4t} \cos 12t & t \geq 0 \end{aligned}$$

2. Find Laplace transform for

$$\begin{aligned} f(t) &= 0 & t < 0 \\ f(t) &= \sin\left(4t + \frac{\pi}{3}\right) & t \geq 0 \end{aligned}$$

Solution

0.1.1 Part a

This is of the form $e^{-at}f(t)$, hence use the property of Laplace transform

$$\mathcal{L}(e^{-at}f(t)) = F(s+a) \quad (1)$$

Where $F(s)$ is Laplace transform of $f(t)$. But $\mathcal{L}(\cos \omega t) = \frac{s}{s^2 + \omega^2}$, therefore

$$F(s) = \mathcal{L}(\cos 12t) = \frac{s}{s^2 + 144}$$

Hence (1) becomes

$$\begin{aligned} \mathcal{L}(e^{-at}f(t)) &= \mathcal{L}(e^{-0.4t} \cos 12t) \\ &= F(s+a) \\ &= \frac{(s+0.4)}{(s+0.4)^2 + 144} \end{aligned}$$

0.1.2 Part b

I can not solve $\mathcal{L}\left(\sin\left(4t + \frac{\pi}{3}\right)\right)$ by using the property that

$$\mathcal{L}(f(t-a)) = e^{-as}F(s)$$

Because here delay $\frac{\pi}{3} > 0$ where the above property is valid for $a < 0$. Instead, writing

$$\begin{aligned} \sin(\omega t + \theta) &= \sin(\omega t) \cos \theta + \cos(\omega t) \sin \theta \\ \sin\left(4t + \frac{\pi}{3}\right) &= \sin(4t) \cos \frac{\pi}{3} + \cos(4t) \sin \frac{\pi}{3} \\ \mathcal{L}\left(\sin\left(4t + \frac{\pi}{3}\right)\right) &= \cos \frac{\pi}{3} \mathcal{L}(\sin 4t) + \sin \frac{\pi}{3} \mathcal{L}(\cos 4t) \end{aligned} \quad (2)$$

But $\cos \frac{\pi}{3} = \frac{1}{2}$ and $\sin \frac{\pi}{3} = \sqrt{3}$ and $\mathcal{L}(\sin \omega t) = \frac{\omega}{s^2 + \omega^2} \implies \mathcal{L}(\sin 4t) = \frac{4}{s^2 + 16}$ and $\mathcal{L}(\cos \omega t) = \frac{s}{s^2 + \omega^2} \implies \mathcal{L}(\cos 4t) = \frac{s}{s^2 + 16}$. Hence substituting into eq (2) gives

$$\begin{aligned}\mathcal{L}\left(\sin\left(4t + \frac{\pi}{3}\right)\right) &= \frac{1}{2} \frac{4}{s^2 + 16} + \sqrt{3} \frac{s}{s^2 + 16} \\ &= \frac{1}{2} \left(\frac{4 + \sqrt{3}s}{s^2 + 16} \right)\end{aligned}$$