90 + 100

Part 2: Midterm

Problem 1 (10 Pts Extra Credit)

An oscilloscope is used to measure this:
Answer b   a) resistance b) voltage c) current d) power

The time constant of a first-order system tells when the output has gotten how far along the way to its final value?

Answer c   a) 37% b) 10% c) 63% d) 90%

If you put a sine wave into a linear system, you get the following out
Answer d   a) square wave
b) sine wave at different frequency
c) triangle wave
d) sine wave at same frequency, scaled and shifted

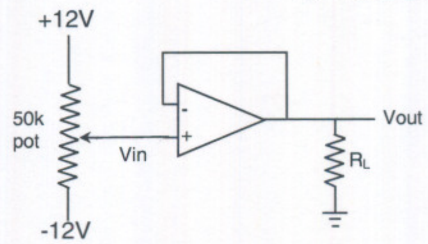
A filter scales a sinusoidal input. The amount of scaling is determined by:
Answer a   a) the magnitude of the transfer function, evaluated at $s=j\omega$
b) the magnitude of the transfer function, evaluated at $s = \omega$
c) the phase of the transfer function, evaluated at $s=j\omega$

A low pass filter attenuates
Answer b   a) low frequencies
b) high frequencies
c) a band of frequencies

K

Problem 2 (25 pts)

How close is V_{out} to V_{in} for the following voltage follower circuit, if the op-amp gain is 1,000? (Hint, use the fact that $V_o = K(V_+ - V_-)$ for the op amp)



$V_o = K(V_+ - V_-)$

since negative feed back

$V_o = V_-$

so $V_o = K(V_+ - V_o) \Rightarrow V_o = KV_+ - KV_o$

$\Rightarrow V_o(1+K) = KV_+$

i.e. $V_o(1+K) = KV_{in}$

so $\dfrac{V_o}{V_{in}} = \dfrac{K}{1+K} = \dfrac{1000}{1001} \approx \boxed{1}$

so V_o almost same as V_{in}.

Problem 3 (25 pts)

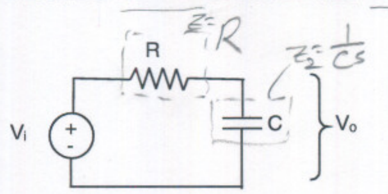
How does the following circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\dfrac{1}{2\pi} = 0.16$ Hz.

Assume R = 1 kiloohm and C=1milliFarad.

10^3 10^{-3}

$V = Ri$

$i = \frac{V}{R}$



$V_i - Ri - V_o = 0$

$V_i - R\left(C\dfrac{dV_o}{dt}\right) - V_o = 0$

$q = CV$

$\dfrac{dq}{dt} = i = C\dfrac{dv}{dt}$

$\boxed{I = Cs\,V(s)}$

so $\boxed{V(s) = I\dfrac{1}{Cs}}$

$RC\dfrac{dV_o}{dt} + V_o = V_i \Rightarrow \dfrac{dV_o}{dt} + V_o\dfrac{1}{RC} = \dfrac{V_i}{RC}$

-10 $\rightarrow \boxed{V_o = V_i\left(1 - e^{-\frac{t}{\tau}}\right)}$ where $\tau = RC$

To find transfer function use impedance

$V_i(s) - Z_1 I(s) - V_o(s) = 0$

but $V_o(s) = \dfrac{1}{Cs} I(s) \Rightarrow I(s) = V_o(s)Cs$

so $V_i(s) - Z_1 V_o(s)Cs - V_o(s) = 0 \Rightarrow V_i(s) = V_o(s)(1 + RCs)$

$\dfrac{V_o}{V_{in}} = \dfrac{1}{1+RCs} \Rightarrow \boxed{H(s) = \dfrac{1}{1+RCs}} \Rightarrow H(j\omega) = \dfrac{1}{1+RCj\omega}$

$RC\omega$

1

as $\omega \to \infty$, $|H(j\omega)| \to 0$

as $\omega \to 0$, $|H(j\omega)| \to 1$

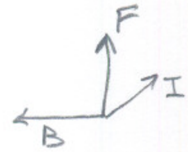
so $\boxed{\text{Low pass}}$ when $\omega = \dfrac{1}{2\pi}$

$|H(j\omega)| = \dfrac{1}{\sqrt{1+(RC\omega)^2}} = \dfrac{1}{\sqrt{1+\left(\frac{1}{2\pi}\right)^2}} = \boxed{\dfrac{1}{\sqrt{1+\frac{1}{4\pi^2}}}}$ ≈ 1

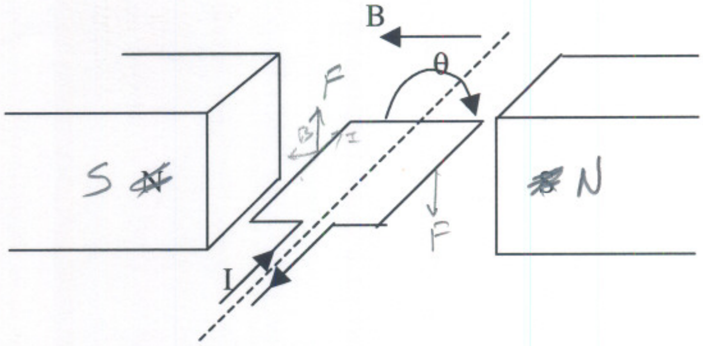
$\angle H(j\omega) = 0 - \tan^{-1} RC\omega$

$= \boxed{-\tan^{-1}\frac{1}{2\pi}}$

Problem 4: 25 pts

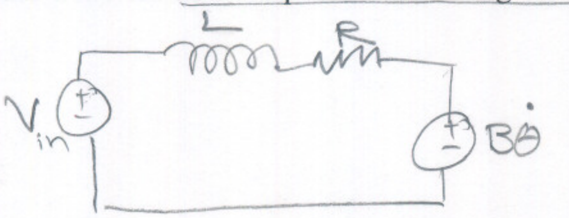


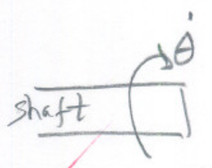
a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.



$\theta = 90^\circ$

b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:





motor side

$$\boxed{V_{in} - L\frac{di}{dt} - Ri - B\dot{\theta} = 0}$$

$$\boxed{i = \frac{\tau}{B}}$$ mechanical side

$\tau = Bi$

$\boxed{i = \frac{\tau}{B}}$

b. Solve this differential equation for the current through the motor as a function of time when:
- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

when shaft fixed $\Rightarrow \dot{\theta} = 0$.

so $V_{in} - L\frac{di}{dt} - Ri = 0$.

so $L\frac{di}{dt} + Ri = V_{in} \Rightarrow \boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V_{in}}{L}}$

homogeneous: $i(t) = Ae^{-\frac{R}{L}t}$, particular $\boxed{i = \frac{V_{in}}{R}}$

so $\boxed{i(t) = Ae^{-\frac{R}{L}t} + \frac{V_{in}}{R}}$ at $t=0$, $i(t)=0 \Rightarrow 0 = A + \frac{V_{in}}{R} \Rightarrow A = -\frac{V_{in}}{R}$

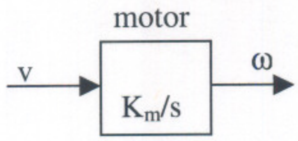
so $i(t) = V_{in}\left(\frac{1}{R} - \frac{1}{R}e^{-\frac{R}{L}t}\right) = \boxed{\frac{V_{in}}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$

so $\boxed{i(t) = \frac{V_{in}}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$ so $\tau = \frac{B}{R}V_{in}\left(1 - e^{-\frac{R}{L}t}\right)$

at steady state $\tau = \frac{B}{R}V_{in}$

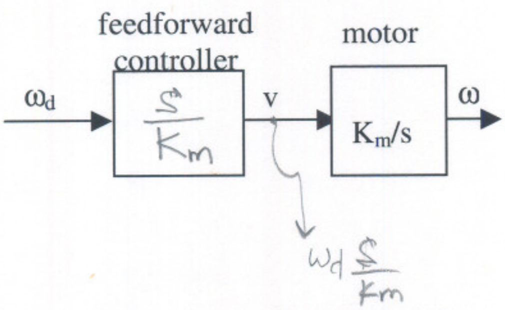
Problem 5: 25 pts

1) You want to control the speed of a motor. You are using a current amplifier with the motor, so the speed is related to the input voltage to the current amplifier by the following transfer function:



where v is the voltage input to the motor and ω is the angular velocity of the shaft and K_m is a constant.

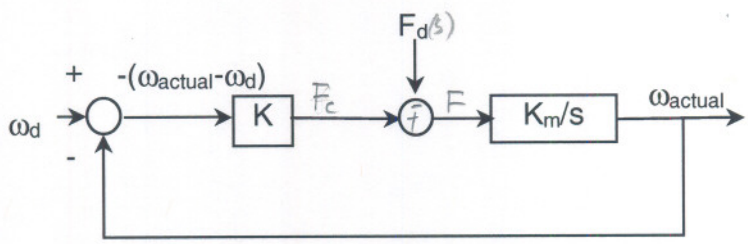
a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What transfer function should the controller box have to make the output equal the desired output? Write this function controller box.



since $V = w_d \frac{s}{K_m}$

so $w = V \frac{K_m}{s} = w_d \frac{s}{K_m} \cdot \frac{K_m}{s}$
$= w_d$

b) One of the major benefits of feedback is its ability to cancel the effects of unmodeled "disturbances". Assume you build a feedback controller, but there is a disturbance force F_d affecting the motor:



Derive an expression that relates ω_{actual} to ω_d and F_d, then prove that the disturbance is cancelled if K is large enough.

$F_c = K(\omega_d - W_a)$, $F = F_c - F_d = k(\omega_d - W_a) - F_d$

$W_{actual} = F \frac{K_m}{s} = [K(\omega_d - W_a) - F_d] \frac{K_m}{s}$

$W_a = [K\omega_d - KW_a - F_d] \frac{K_m}{s} = \frac{KK_m}{s} \omega_d - \frac{KK_m}{s} W_a - \frac{K_m}{s} F_d$

$\boxed{W_a \left[1 + \frac{KK_m}{s}\right] = \frac{KK_m}{s} w_d + \frac{K_m}{s} F_d}$

when $K \gg K_m$ Then $\frac{KK_m}{s} \gg \frac{K_m}{s}$

So $\boxed{W_a \approx W_d}$

since $\frac{K_m}{s}$ is small in comparison to weights on W_a and W_d..

so effect of F_d is minimized.