MAE 106 Mechanical Systems Laboratory
Winter 2005 Design Exam

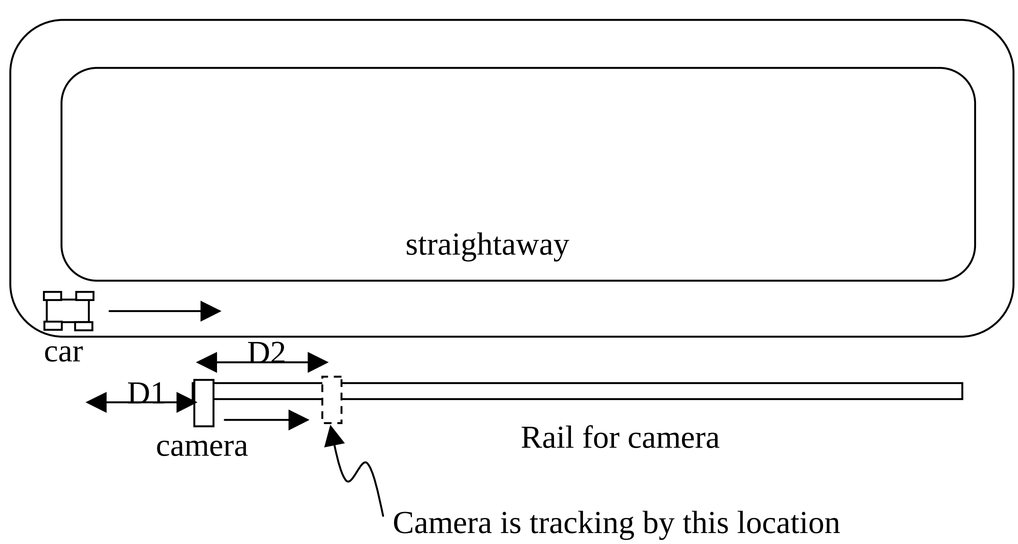
You have been hired as a control engineering consultant for NASCAR, the car racing organization. Your job is to design a control circuit for a robotic camera that will ride on a rail along the straightaway (i.e. the straight part of the racetrack). The camera control system will receive a measurement of the velocity from a selected car. The velocity information will come from the car's own speedometer, via a wireless link. Your job is to make the camera track the car, so that television viewers worldwide can "ride along" next to the driver on the straightaway. You are given the following information:

- The motor that drives the robotic camera along its track is a DC brushed motor with a current amplifier. It produces a force on the camera in response to a voltage input with a calibration coefficient of C_1 N/volt.
- The robotic camera has a mass of M.
- There is a tachometer on the robotic camera, which returns the linear velocity of the camera as a voltage with a calibration coefficient of C_2 volts/meter/sec
- The robotic rail system has some static friction, which can be modeled as constant force F_f. Dynamic friction is negligible.
- The wireless link begins working when the car is D_1 meters away from the camera. The robot should be moving at the right speed, right next to the car, after it moves D_2 meters.
- You may assume that the car moves at a constant velocity in the straightaway, and the maximum velocity that it moves at is S.
- You are to design the controller with the lowest gains possible, because NASCAR plans to update your controller next year using a computer-based system. They believe the computer-based system will work better if the controller you design has as low-as-possible gains, because of possible time delays in sampling with the planned computer-based system.

Design an analog circuit to control the robotic camera. You will get full credit if you:

1) Show your control law, in MKS units.
2) Provide appropriate control gains with units.
3) Show your control law, in units of volts
4) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
5) Choose appropriate values for the resistors and capacitors in your circuit.

PLEASE ENTER YOUR ANSWERS ON THE ANSWER SHEET



velocity is the signal that is available. Implementing position control would require a position sensor or an integrator.

$-K_p(V-V_d) - K_d\dot V = m\dot V$

$-K_p(V+V_d) = (m+K_d)\dot V$ 1st order

Answer Sheet NAME: SOLUTION

1) Control law; MKS units (20 pts)

$F = -K_p(v-v_d) - K_I\int(v-v_d)dt$

Basic idea: Use proportional velocity feedback (15 pts)

- Adding an integral term eliminates steady state error from friction (5 pts)
- Derivative term is not necessary - does not increase order of system or improve performance

→ To find control gains, find closed loop dynamics

$m\dot v = F_f - K_p(v-v_d) - K_I\int(v-v_d)dt$ } $\frac{d}{dt}$

$m\ddot v + K_p\dot v + K_I v = K_I v_d$ 2nd order system

$$\frac{V}{V_d} = \frac{K_I}{ms^2+K_ps+K_I} = \frac{\frac{K_I}{m}}{s^2+\frac{K_p}{m}s+\frac{K_I}{m}} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

If just P ctrl:

$-K_p(v-v_d) = m\dot v$; step input

$V = V_d(1-e^{-t/\tau})$ $\tau = \frac{m}{K_p}$

want $3\tau = T = \frac{D}{S}$ $D = D_1+D_2$

$\tau = \frac{T}{3} = \frac{m}{K_p} \Rightarrow K_p = \frac{3m}{T}$

2) Control gains (with units) (20 pts)

$K_I = \frac{9mS^2}{D^2}\left[\frac{N}{m}\right]$ $K_p = \frac{3mS}{D}\left[\frac{Ns}{m}\right]$ if just P-control

or

$K_p = \frac{6mS}{D}\left[\frac{Ns}{m}\right]$

$D = D_1 + D_2$

System needs to reach V_d after $T = \frac{D}{S}$ seconds $D = D_1+D_2$

Use 5% criterion $3\tau = \frac{3}{\omega_n\zeta}$ (we'll set $\zeta = 1$) $\Rightarrow \frac{3}{\omega_n} = T$ $\omega_n = \frac{3}{T} = \frac{3S}{D}$

$\omega_n^2 = \frac{K_I}{m} \Rightarrow K_I = m\omega_n^2 = \frac{9mS^2}{D^2}\left[\frac{N}{m}\right]$

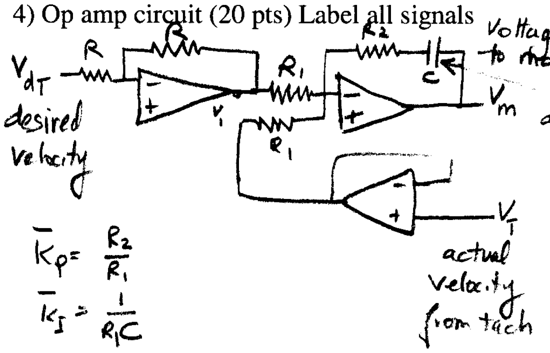
$\zeta = \frac{K_p}{2\sqrt{K_I m}} \Rightarrow K_p = 2\sqrt{K_I m}\,\zeta = 2\sqrt{\frac{9m^2S^2}{D^2}} = \frac{6mS}{D}\left[\frac{Ns}{m}\right]$

Note: Input is a step input since transmitter suddenly starts working

3) Control Law; Volts as units (20 pts)

$$V_m = \underbrace{\frac{-K_p}{C_1C_2}}_{\bar K_p}(V_T - V_{dT}) - \underbrace{\frac{K_I}{C_1C_2}}_{\bar K_I}\int(V_T - V_{dT})dt$$

4) Op amp circuit (20 pts) Label all signals



$\bar K_p = \frac{R_2}{R_1}$

$\bar K_I = \frac{1}{R_1C}$

$\frac{V_{dT}}{R} = \frac{-V_1}{R} \Rightarrow V_1 = -V_{dT}$

$\frac{V_1}{R_1} + \frac{V_T}{R_1} = \frac{-V_m}{R_2 + \frac{1}{sC}}$

$\frac{1}{R_1}(V_T - V_{dT}) = \frac{-V_m sC}{R_2Cs+1}$

$V_m = -\frac{(R_2Cs+1)}{R_1sC}(V_T - V_{dT})$

$V_m = -\left(\frac{R_2}{R_1} + \frac{1}{R_1Cs}\right)(V_T - V_{dT})$

$= -\left(K_p + K_I\frac{1}{s}\right)(V_T - V_{dT})$

Express in units of volts

$F = C_1V_m$ V_m = motor voltage

$V_T = C_2V$ V_T = tach voltage

$C_1V_m = -K_p\left(\frac{V_T}{C_2} - \frac{V_{dT}}{C_2}\right)$

$V_m = \frac{-K_p}{C_1C_2}(V_T - V_{dT})$

5) Resistor and Capacitor Values (20 pts)

$R_1 = \frac{C_1C_2}{K_IC} = \frac{C_1C_2D^2}{9mS^2C}$ choose C arbitrarily

$R_2 = \frac{R_1K_p}{C_1C_2} = \frac{R_1 6mS}{C_1C_2D}$

$\bar K_I = \frac{1}{R_1C} \Rightarrow R_1 = \frac{1}{\bar K_I C} = \frac{C_1C_2}{K_IC}$ C arbitrary

$\bar K_p = \frac{R_2}{R_1} \Rightarrow R_2 = R_1\bar K_p = \frac{R_1K_p}{C_1C_2}$