MAE 106 Midterm Exam: Closed Book/Notes Section
Winter 2004

Mean = 37/50
$\sigma = 8$

University of California, Irvine
Department of Mechanical and Aerospace Engineering

4 1. What is Ohm's Law?

$V = IR$

3 2. What is Kirchoff's current law?

$\sum_{node} i = 0$

3 3. What is Kirchoff's voltage law?

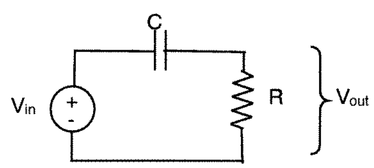
$\sum_{loop} V = 0$

4 4. If you input a sinusoidal input u(t) = asin(ωt) into a linear, time-invariant system, with a transfer function H(s), what is the output x(t)?

$x(t) = a|H(j\omega)|\sin(\omega t + \phi_H(j\omega))$

8 5. Now, apply these concepts to find the frequency response of the following circuit. Provide both the magnitude and phase response.

−2 low pass but right method



Transfer function:

$V_{out}(s) = \frac{R}{R + \frac{1}{sC}} V_{in}(s)$ or ODE 2pts

$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{RCs}{1 + RCs}$

Magnitude response $= |H(j\omega)| = \left|\frac{RCj\omega}{1 + RC\omega j}\right| = \frac{RC\omega}{\sqrt{1 + (RC\omega)^2}}$ 3

Phase response $= \phi_H(j\omega) = \tan^{-1}\frac{RC\omega}{0} - \tan^{-1}\frac{RC\omega}{1}$ 3

$= 90^\circ - \tan^{-1}RC\omega$

4 6. What type of filter is this? Provide mathematical proof.

if $\omega \to 0$ $|H(j\omega)| = \frac{0}{\sqrt{1+0}} = 0$ attenuates low frequencies

3 if $\omega \gg \frac{1}{RC}$ $|H(j\omega)| \approx \frac{RC\omega}{RC\omega} = 1$ passes high frequencies

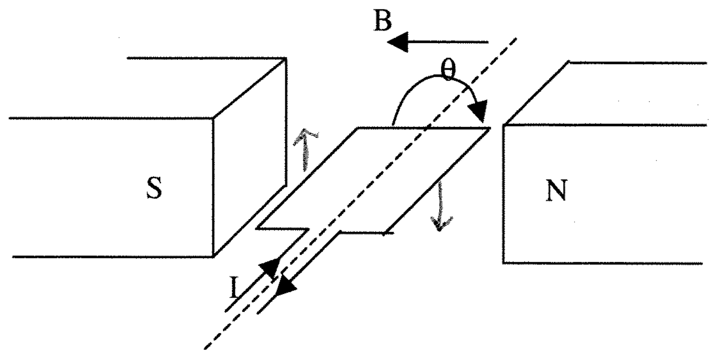
1 ∴ High pass filter

26 Low pass but right method −1

4 7. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.

$\vec{F} = i\vec{l} \times \vec{B}$

$\theta = 90^{\circ}$



8 8. Assume that the commutation is working correctly, and the motor's torque constant is B. Find the torque that the motor produces as a function of time, when:

- The shaft of the motor is held fixed
- A constant voltage is applied across the motor at time = 0
- The initial current through the motor is zero

2 motor eqn: $L\frac{di}{dt} + B\dot{\theta} + iR = V$ $B\dot{\theta} = 0$ because shaft is fixed

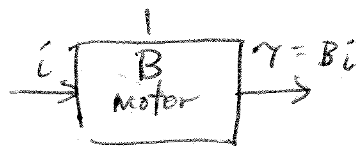
2 Homog. soln: $i = Ae^{-t/\tau_c}$ $\tau_c = \frac{L}{R}$

$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$ 2 Part soln: $i = V/R$

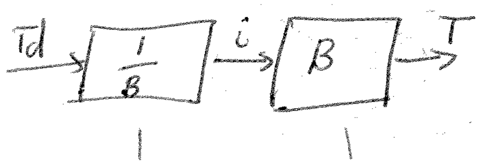
2 Tot soln: $i = V/R + Ae^{-t/\tau_c}$ but $i(0) = 0$ so $i = \frac{V}{R}(1 - e^{-t/\tau_c})$ $\tau_c = \frac{L}{R}$

$\tau = Bi = \frac{BV}{R}(1 - e^{-t/\tau_c})$

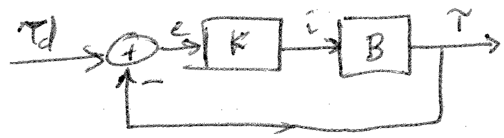
2 9. Draw a block diagram of the motor, assuming that the input is the current i and the output is torque τ.



4 10. Draw a block diagram of an open-loop (i.e. feedforward) controller for the plant of part (9), where the input to the controller is τ_d, the desired torque output of the motor.



4 11. Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal. Again, the input to the controller should be τ_d, the desired torque output of the motor.



2 12. What type of sensor do you need to make this feedback controller work?

torque sensor

24

MAE 106 Midterm Exam: Open Book/Notes Section
Winter 2004

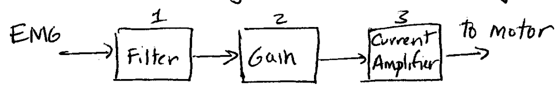
University of California, Irvine

Department of Mechanical and Aerospace Engineering

Your goal is to design a power-assist for a motorized bicycle. The device will measure the small voltage produced by leg muscles of the rider, using small electrodes taped over the muscles. This measurement is called the electromyogram, or “EMG”. The device will low-pass filter the EMG voltage signal with a cutoff frequency of 10 Hz in order to get a smooth control signal, and generate a motor torque proportional to the low-pass-filtered EMG signal. The proportionality constant should be adjustable between two values by flipping a switch, from 10 (workout mode) to 100 (cruise mode). Thus, when the rider pedals, he or she will also activate the motor attached to the bicycle, giving a power assist.

Design a circuit using operational amplifiers that can implement the power-assist controller for the bicycle. You may assume that you have a power operational amplifier capable of generating the current needed to directly power the motor. Hint: In order to control the current through the motor, you can control the voltage across a resistor that is in series with the motor, using the power op amp.

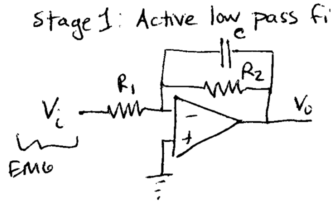
I choose to design a circuit with 3 stages:



Mean = 28/50

$\sigma = 10$

Stage 1: Active low pass filter (could also use passive filter w/ a buffer)



$$\frac{V_o}{V_i} = \frac{-R_2}{R_1}\left(\frac{1}{1+R_2 C s}\right)$$

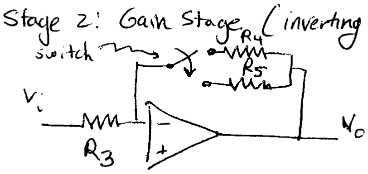
gain, low pass filter

cutoff frequency $\omega_c = \frac{1}{R_2 C} = 2\pi(10\,\text{Hz}) = 63\ \text{rad/sec}$

choose $C = 100\,\mu F \Rightarrow R_2 = 630\,K\Omega$

Choose $R_1 = 630\,K\Omega$ so Gain $= -1$

Stage 2: Gain Stage (inverting amplifier; could also use non-inverting amplifier)



By flipping the switch, you can change the gain, for cruise or workout mode

$$\frac{V_o}{V_i} = \frac{-R_4}{R_3}$$

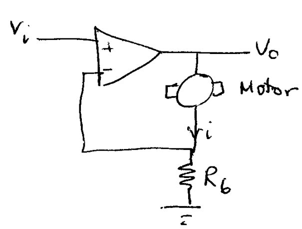
Cruise Mode: Switch up, gain = 100

choose $R_3 = 1\,K\Omega$, $R_4 = 100\,K\Omega$

Workout Mode: Switch down, gain = 10

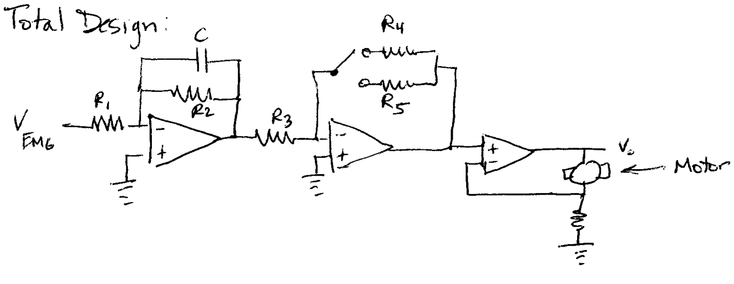
$\Rightarrow R_4 = 10\,K\Omega$

Stage 3: Current Amplifier



Note: $i = \frac{V_i}{R_6} \Rightarrow$ current through motor is proportional to input voltage. Choose R_6 to give appropriate levels of current for motor. R_6 must be a power resistor, + this op-amp must be a power op-amp, both capable of large currents

Total Design:



Have fun riding!

Grading:

1) 10 pts Filter circuit
2) 10 pts R, C values for ω_c
3) 5 pts Buffering OK
4) 10 pts Gain stage
5) 15 pts Current amplifier