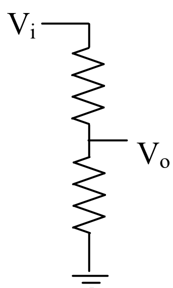
MAE 106 Midterm Exam
Winter 2003

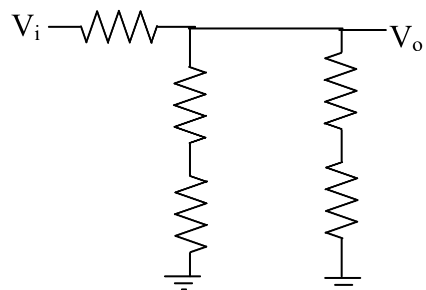
University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1 Circuits (25 pts)

a) Find V_o for the following two circuits. Assume all resistors values = R.

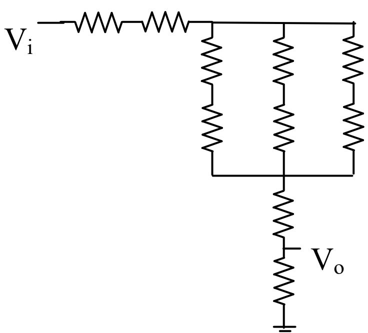


Answer:



Answer:

b) For the following circuit, draw conceptual sketches to show how you would simplify the network to solve for V_o. You do not need to find V_o, just illustrate the steps.



c) What are the two “golden rules” of op-amp analysis?

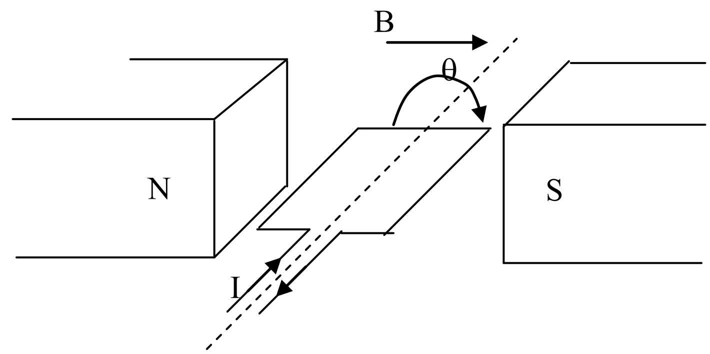
d) What two conditions must be true for these golden rules to apply?

e) Using an op-amp and resistors, design a circuit to amplify an input signal by a factor of 20. Show mathematically that your design works.

f) Assume that you have a low-power control signal from a computer, and that you would like to make a motor spin when the control signal is +5 v, and to stop spinning when the control signal is 0 V. Design a circuit using a MOSFET to achieve this control.

Problem 2: Motors (25 pts)

a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest?
Assume the armature is initially at θ = 0° as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.



b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:

b. Solve this differential equation for the current through the motor as a function of time when:

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

c. Plot the torque that the motor generates as a function of time for the conditions described in part b. Label the axes, the final value of the torque, and the time at which the torque has reached 63% of its final value. Assume the motor's torque constant is some constant B, equal to the back EMF constant.

d. What is the term for the maximum torque a motor can produce when its shaft is held constant?

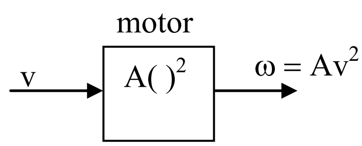
e. What is the term for the maximum speed that a motor free to spin will reach?

f. What happens to an unloaded motor spinning at steady-state speed ω when you double its input voltage?

g. At what speed does an ideal DC brushed motor produce maximum power and why?

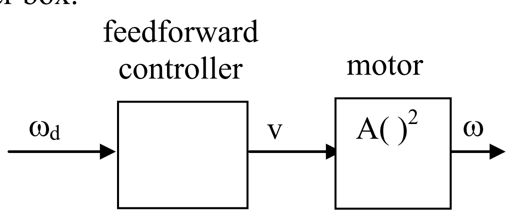
Problem 3: Control Theory (25 pts)

1) You design a new type of motor for which the speed is proportional to the input voltage squared:



where v is the voltage input to the motor and ω is the angular velocity of the shaft and A is a constant.

a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What function should the controller box compute to make the output equal the desired output? Write this function controller box.



b) If the estimate of A used by the feedforward controller is too small by 10%, how much faster will the actual speed be than the desired speed?

c) Draw a block diagram of a feedback controller for the motor, label all arrows, including the error signal.

d) Prove that, the actual velocity equals the desired velocity as the feedback gain gets large.

Problem 4: Frequency Analysis: Motivation, Theory, and Practice (25 pts)

a) Give two reasons for studying frequency domain analysis:

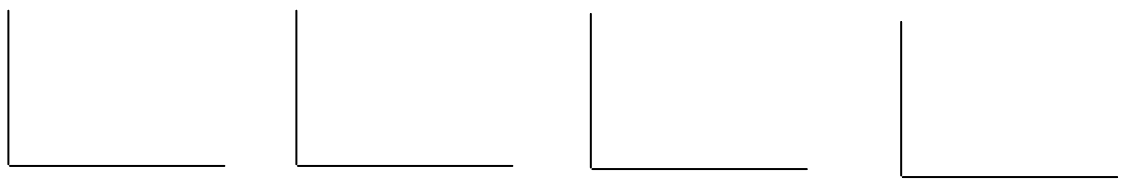
g) If you input a sinusoidal input u(t) = asin(ωt) into a linear, time-invariant system, what is the output x(t)? Express your answer in terms of an equation and in words.

h) Below is a proof of the correct answer to part b). On the left are the equations for each step of the proof. On the right is a description of what is happening in the proof. Fill in all blanks to complete the proof.

Step	Description	Equation
1	An n^{th} order linear system with output x(t) and input u(t) can be described by a differential equation like this:	
2		X(s)=B(s)/A(s)U(s)+IC(s)/A(s)
3	Assume the system is stable, then in the steady state Equation 2 can be simplified to:	
4	The Laplace transform of the sinusoidal input is:	
5	Thus, the steady state output of the system in the frequency domain is:	
6		$K_1/(s+jw)+K_2/(s-jw)+K_3/(s-s1)+K_4(s-s2)+\ldots$
7	But the system is stable, so the K3 and K4 terms go to zero with time, and the output in the time domain is thus Equation 7, with $K_1=-a/(2j)G(-jw)$, and $K_2 = a/(2j)G(jw)$, G(s)=B(s)/A(s)	
8	Simplifying Equation 7 using the fact $\sin(\theta) = 1/2j(e^{j\theta}-e^{-j\theta})$ gives:	
	PROOF COMPLETE	PROOF COMPLETE

i) What two pieces of information do you need to describe the frequency response of a system?

j) Draw an example of the magnitude response of the following types of filters. Make sure to label the axes.



Low Pass Filter | High Pass Filter | Notch Filter | Band-pass filter

k) Prove that a mass M acts like a low-pass filter, if force f is considered its input and position x its output