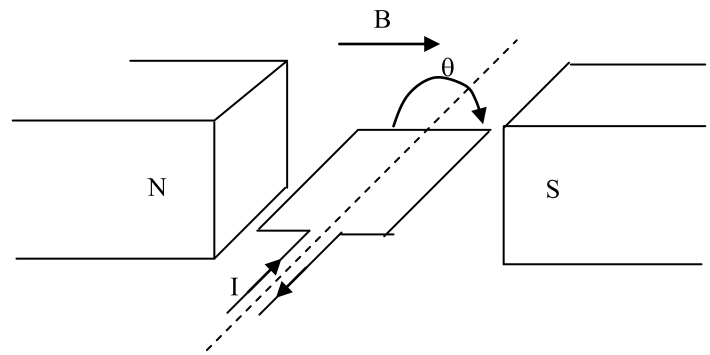
MAE 106 Midterm Exam
Winter 2001

University of California, Irvine
Department of Mechanical and Aerospace Engineering

Problem 1: Motors (25 pts)



a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at $\theta = 0°$ as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.

b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:

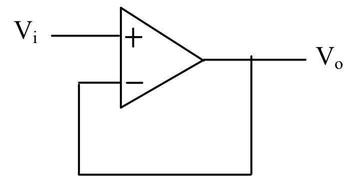
c. Derive and plot the torque versus velocity relationship for the motor. Plot torque on the y-axis and velocity on the x-axis. Label the x-axis and y-axis intercepts with the appropriate terminology. Assume that:
- the motor receives a constant voltage input of V
- the inertia of the motor shaft equals J
- the motor's torque constant is B and is equal to the back-EMF constant
- the current into the motor does not change.

d. For what torque load does the motor produce maximum power?

e. Assume you have a low-power control signal from a computer, and that you would like the unloaded motor shaft to spin at ω_d when the control signal = +5V, and to stop spinning when the control signal is 0 V. Design a circuit using only a MOSFET, a power supply, and resistors to achieve this control. Label the gate, drain, and source of the MOSFET, and the control input from the computer. Specify the voltage of the power supply assuming the motor's torque constant is B and is equal to the back-EMF constant.

Problem 2: Op Amps and Feedback (25 pts)

a. Write the name of this circuit:



b. Briefly describe a practical situation in which you might want to use the circuit in a)

c. Assume that $V_o = K(V_+ - V_-)$ for the op amp. How big would K have to be such that V_o is within 1% of V_i?

d. For a real op amp, the output takes some time to respond to the inputs due to delays in the op-amp's internal circuitry. We can model these dynamics with a differential equation:

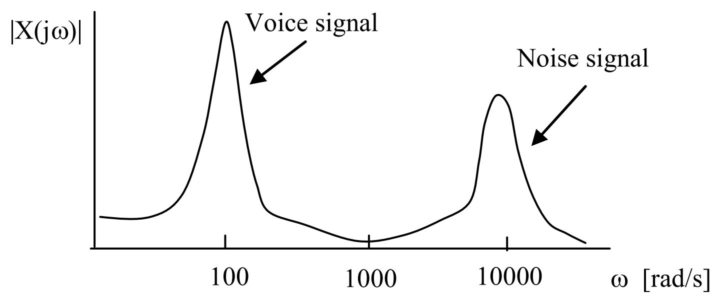
$$\dot{V}_o = -AV_o + AK(V_+ - V_-)$$

where $A > 0$ is a constant. Given these dynamics, plot the voltage response of the circuit in a) to a unit step input in V_i versus time. Label the time at which the output is 63% of its final value.

e. Assume that you misconnect the circuit in a) such that the feedback goes to the non-inverting input instead of the inverting input (i.e. positive feedback instead of negative feedback), and the input goes to the inverting input. Assuming the op-amp has the dynamics in d), for what values of K will the circuit be unstable (i.e. for what values of K will the output go to infinity as time goes to infinity)?

Problem 3: Filters and Signal Processing (25 pts)

You are hired to improve a wire-tapping signal for the FBI. Shown below is a Bode plot of the typical frequency content of the signal that the FBI's currently-used system generates:



a) Sketch (very roughly) what the signal would like in the time domain. Include a time scale.

b) What type of filter should you apply to clean up the signal?

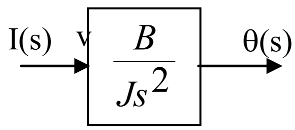
c) You have only a few resistors and capacitors in your toolbox, as shown below. Design an RC circuit to filter the unwanted noise. Draw the circuit, with the input and outputs labeled, and identify the values of R and C that you would use.

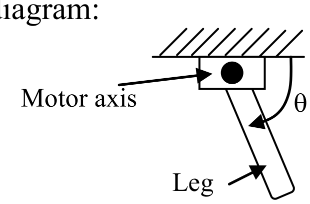
Toolbox Contents: R = 1Ω, 10Ω, 100Ω, 1 KΩ C = 1 pF, 0.1 μF, 1 μF

d) By what factor would your filter attenuate the noise in the signal at 10000 rad/s?

Problem 4: Control Theory and Laplace Transforms (25 pts)

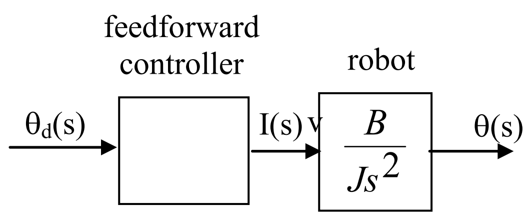
Consider the problem of controlling the angle of a one degree-of-freedom robotic leg. The transfer function of the robot arm is given by the following block diagram:





where I(s) is the current input to the robot's motor, J is the inertia of the motor, B is the motor torque constant, and θ is the angular position of the robot.

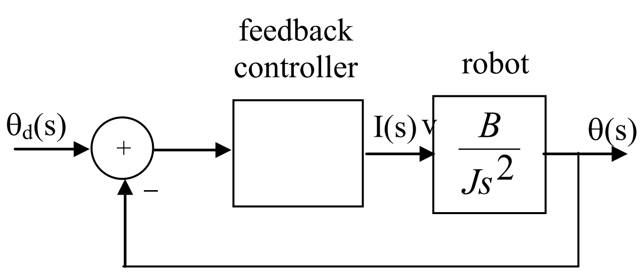
a) Shown to the side is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where θ_d is the desired angular position of the robot. What transfer function should the controller box have to make the output equal the desired output? Write the transfer function the controller box.



b) In the *time domain*, write the relationship between I and θ_d

c) What is a disadvantage of an open-loop controller like this one?

d) A proportional-derivative (PD) feedback controller is a common type of controller that provides a control signal to the plant that is the sum of a signal proportional (by a gain K_p) to the output error and a signal proportional by (a gain K_d) to the derivative of the output error. In the following diagram, fill in the appropriate transfer function such that the robot would be controlled by a PD controller.



e) Find the overall transfer function for the PD-feedback controlled robot system in d)