MAE 106 Mechanical Systems Laboratory
Winter 2004 Design Exam

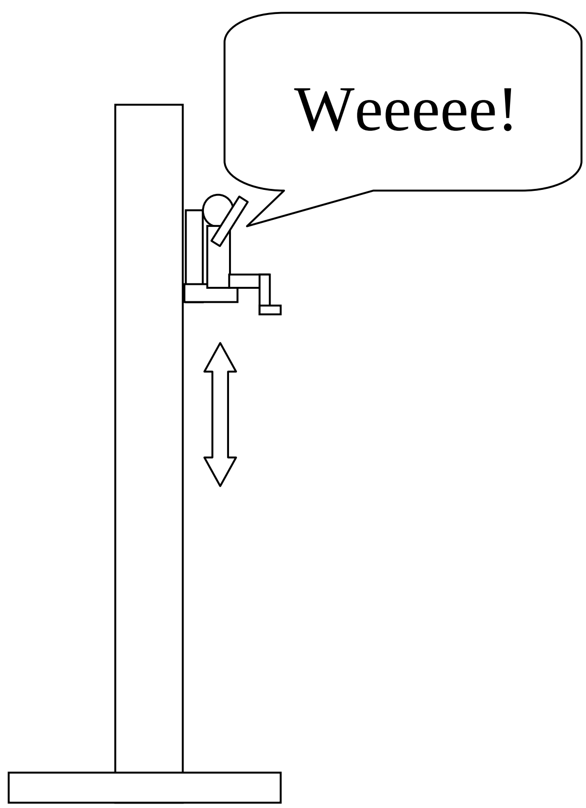
You have been hired as a control engineering consultant for a new ride for Disney's California Adventure Theme Park, called "Bouncing Over California". In this ride, the rider will feel like they are bungee jumping off of different bridges in California. Your job is to design a control circuit for the ride. You are given the following information:

- The ride is in a 100 foot tower that has a linear track. The rider sits in a harness which slides up and down the track. The track is driven by a motor.
- The track has substantial friction, which can be modeled as viscous friction with a damping coefficient of B (i.e. friction force = F=-Bv = damping coefficient*velocity).
- The motor that drives the track is a DC brushed motor with a current amplifier. It produces a force on the harness in response to a voltage input with a calibration coefficient of C_1 N/volt.
- There is a position sensor on the track that measures the position of the harness. The sensor has a calibration coefficient of C_2 volts/meter, and gives a reading of zero volts at the top of the track, where the person enters the harness.
- A typical bungee cord acts like a linear spring with a stiffness of K_b.
- The Disney people would like the system to behave like an undamped spring.

Design an analog circuit to control the security robot. You will get full credit if you:

1) Show your control law, in MKS units.
2) Provide appropriate control gains with units.
3) Show your control law, in units of volts
4) Draw an op-amp circuit that can implement your controller. Label what the inputs and outputs of your circuit should be connected to.
5) Choose appropriate values for the resistors and capacitors in your circuit.

PLEASE ENTER YOUR ANSWERS ON THE ANSWER SHEET



Solutions to MAE106 Winter 2004 Design Exam

1) Answer: (20 pts)

$F = -K_p(x - x_d) - K_d\dot{x}$

2) Answer: (20 pts)

Approach 1: $K_p = K_b$ N/m or kg/s^2

$K_d = -B$ Ns/m or kg/s

Approach 2: $K_p = 100M\omega_s^2$ N/m

$K_d = 2\sqrt{K_p M} - B$ Ns/m

3) Answer: (20 pts)

Note: $F = C_1 v_o$ = v_o = motor voltage

v_s = sensor voltage = $C_2 * x$

v_d = input voltage = $C_2 * x_d$

$$C_1 v_o = -K_p / C_2 (v_s - v_d) - K_d \dot{v}_s / C_2$$

$$v_o = -1/C_1 (K_p / C_2 (v_s - v_d) - K_d \dot{v}_s / C_2)$$

Overview: There are two ways to do this design. In both approaches, we will use a PD position controller to make the controlled system behave like an undamped spring with $K=K_b$. The PD position controller equation is:

$$F = -K_p(x - x_d) - K_d\dot{x}$$

Approach 1: Make the controller have $K_p = K_b$, and cancel the friction with $K_d = -B$. Then the controlled dynamics are:

$$M\ddot{x} + B\dot{x} = -K_p(x - x_d) - K_d\dot{x} = -K_b(x - x_d) + B\dot{x} \rightarrow M\ddot{x} = -K_b(x - x_d)$$

which are the dynamics of an undamped spring with $K=K_b$.
In this approach, we want the rest length of the spring to be half way down the track x_d=50ft =16.5 meters

Approach 2: Make the controller track the motion of an undamped spring with $K=K_b$. In this case, we let x_d=16 (cos(ω_st)-1) meters, so x_d = 0, max(x_d) = 33 m = 100ft. The frequency of the sinusoid ω_s should be the frequency of an undamped spring with $K=K_b$ and a rider of mass M $\omega_s = \sqrt{K_b / M}$.

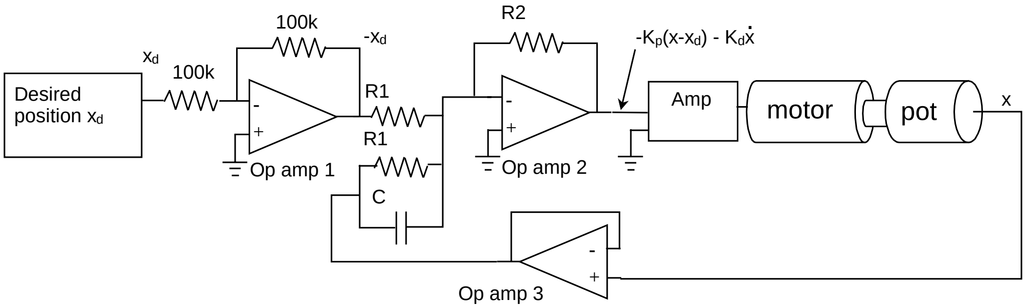
Then, we choose K_p and K_v so that the tracking is good. We choose the bandwidth of the controlled system ω_n = 10 ω_s (the factor 10 is arbitrary; the point is that we want the bandwidth significantly wider than ω_s for good tracking)

$$\omega_n = \sqrt{K_p / M} = 10\omega_s \rightarrow K_p = 100M\omega_s^2 = 100K_b$$

And choose ζ = 1 for good tracking without resonance and find K_d

$$\zeta = (B + K_d)/(2\sqrt{K_p M}) = 1 \rightarrow K_d = 2\sqrt{K_p M} - B$$

4) Answer: (20 pts) The circuit from Lab 6 will work for Approach 2:



For Approach 1, we need to invert the voltage into the capacitor with an op-amp inverter circuit (similar to Op amp 1 circuit), in order to get a negative K_d

5) Answer: (20 pts)

$K_P = R_2/R_1$

$K_d = R_2 C$

So choose:

$R_2 = K_d / C$ $\quad$ $R_1 = R_2 / K_P$