LAB #7 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Friday 2/25/2005 10 AM

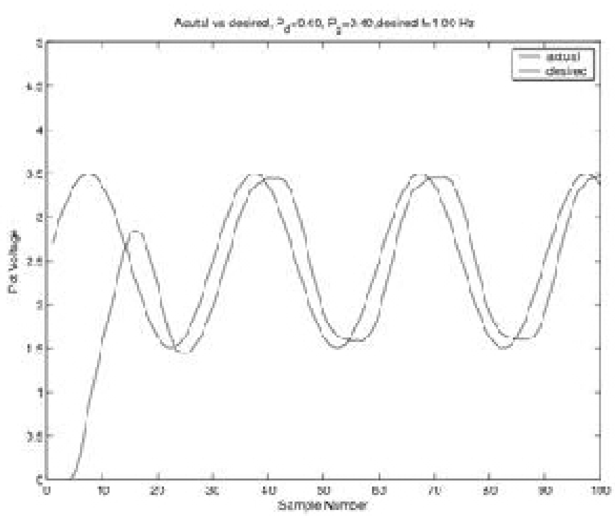
March 2, 2005

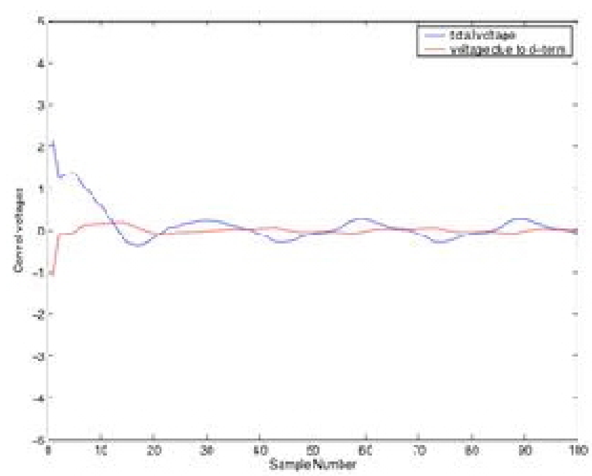
1 Answer 1.

I have tried a number of runs with different proportional and derivative gain constants running at 1 Hz. This plot below shows few of the tests I've run, and below them the one I think achieved the best tracking.

Derivative gain=0.4

Proportional gain=0.4



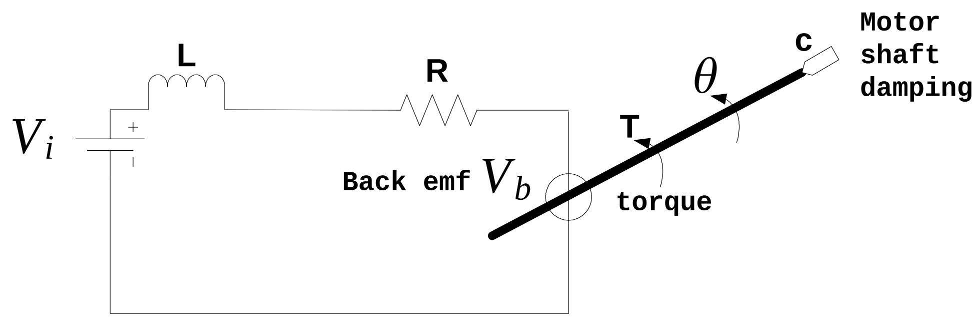


2 Answer 2

First I need to derive the transfer function. I can decide to control the speed of the motor shaft, or its angular position. I need to decide on this since this affect what the transfer function will be. i.e. wither I will select the position or the speed to be the output. In both cases I will take the motor voltage supply as the input.

I selected to use position as the controller variable.

First, I show the model of the motor itself, then the block diagram. Next I show the block diagram with a delay element added in the feedforward path, and the compare the transfer functions with and without delay and show that with delay, it is possible for the output to become unstable.



From the diagram below, using Kirchoff law around the motor circle, we get

$$V_i = L\frac{di}{dt} + Ri + V_b$$

Take Laplace transform we get

$$V_i(s) = sL + RI(s) + V_b(s) \tag{1}$$

Now, we know that the backemf voltage V_b produced is proportional to the angular speed of the shaft. Let this proportionality constant be called B_b then we write

$$V_b = B\frac{d\theta}{dt}$$

Take Laplace transform of the above, we get

$$V_b(s) = B_b s\theta(s) \tag{2}$$

Substitute equation (2) into (1) we get

$$V_i(s) = sL + RI(s) + B_b s\theta(s) \tag{3}$$

Now consider the dynamic equation for the motor shaft, we get

$$T - c\frac{d\theta}{dt} = J\frac{d^2\theta}{dt^2}$$

Where J is the moment of inertial of the motor shaft around its axis of rotation. Take Laplace transform of the above we get

$$T - cs\theta(s) = Js^2\theta(s) \tag{4}$$

We also know that the torque produced is proportional to the current in the motor. Lets call the proportionality constant B_t hence we write

$$T = B_t i \tag{5}$$

Take Laplace transform of (5) we get

$$T = B_t I(s) \tag{6}$$

Substitute (6) into (4) we get

$$\begin{aligned} B_t I(s) - cs\theta(s) &= Js^2\theta(s) \\ I(s) &= \frac{Js^2\theta(s) + cs\theta(s)}{B_t} \\ &= \frac{\theta(s)\left(Js^2 + cs\right)}{B_t} \end{aligned} \tag{7}$$

Now substitute (7) into (3) we get

$$\begin{aligned} V_i(s) &= sL + R\frac{\theta(s)\left(Js^2 + cs\right)}{B_t} + B_b s\theta(s) \\ &= sL + \theta(s)\left[\frac{R\left(Js^2 + cs\right)}{B_t} + sB_b\right] \end{aligned} \tag{8}$$

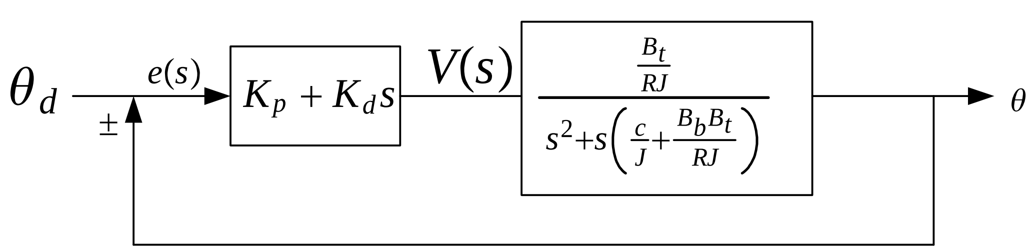
Now L is usually very small compare to R so equation (8) can be written as

$$V_i(s) = \theta(s)\left[\frac{R\left(Js^2 + cs\right)}{B_t} + sB_b\right]$$

Hence the transfer function between V_i and θ is

$$\begin{aligned} \frac{\theta(s)}{V_i(s)} &= \frac{1}{\frac{R(Js^2+cs)}{B_t} + sB_b} = \frac{B_t}{R\left(Js^2 + cs\right) + sB_bB_t} \\ &= \frac{\frac{B_t}{RJ}}{s^2 + s\left(\frac{c}{J} + \frac{B_bB_t}{RJ}\right)} \end{aligned}$$

This transfer function is in the standard form. It is a second order system. The above is the transfer function of the plant itself. Now I put the above into the loopback block diagram, assuming the controller we used is PD controller of the form $K_p + k_d s$ we get this



Now we do block simplification to obtain the closed loop transfer function. Let $P(s) = \frac{\frac{B_t}{RJ}}{s^2+s\left(\frac{c}{J}+\frac{B_b B_t}{RJ}\right)}$, hence the feedforward transfer function is $G_o(s) = (K_p + K_d)P$ hence the closed loop transfer function is $G_c(s) = \frac{G_o}{1+G_o} = \frac{(K_p+K_d)P}{1+(K_p+K_d)P}$

Let $K_p + K_d = K$ then we write $G_c(s) = \frac{KP}{1+KP}$

The charaterstic equation is $1 + KP = 0$. The closed loop poles are the roots of this equation.

Replace P above to be able to solve for the roots, we get

$$\begin{aligned} 1 + K\frac{\frac{B_t}{RJ}}{s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right)} &= 0 \\ s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right) + K\frac{B_t}{RJ} &= 0 \end{aligned}$$

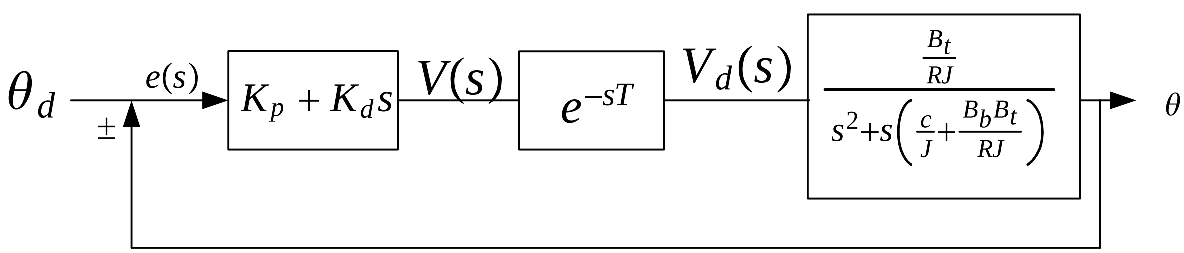
the roots are

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right)}{2} \pm \frac{\sqrt{\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right)^2 - 4K\frac{B_t}{RJ}}}{2}$$

We see from the above, that independent of the values under the $\surd$sign, the system will have its poles in the left hand side. This is because the quantity $\frac{c}{J} + \frac{B_b B_t}{RJ}$ is positive.

Hence the system is always stable no matter how large the gain K is.

Now let see what happens when we add the effect of Labjack into the system, model this effect as a time delay, which in the Laplace transform becomes e^{-sT} where T is the time it takes Labjack to sample one data point, i.e. T is the sampling period. Hence now the block diagram becomes



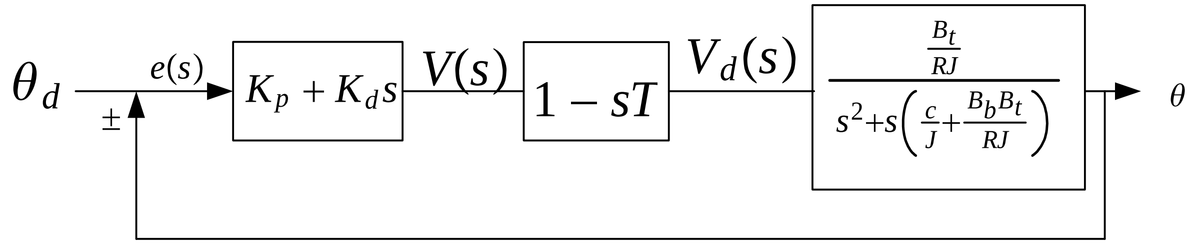
Where I wrote V_d as the output from the labjack. (delayed voltage).

Now, since $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} - \cdots$

Then $e^{-sT} = 1 + (-sT) + \frac{(-sT)^2}{2} + \frac{(-sT)^3}{3!} - \cdots = 1 - sT + \frac{s^2T^2}{2} - \frac{s^3T^3}{3!} - \cdots$

Now for very small T , all terms with s^n for $n > 1$ can be ignored. Hence we get an approximation $\boxed{e^{-sT} = 1 - sT}$

Hence the above system becomes



Now obtain the closed loop transfer function.

The open loop transfer function is $G_o(s) = (K_p + K_d)(1 - sT)P(s)$

As before, let $(K_p + K_d) = K$, hence we get $G_o(s) = K(1 - sT)P(s)$

Then the closed loop transfer function is

$$G_c(s) = \frac{G_o}{1 + G_o} = \frac{K(1 - sT)P(s)}{1 + K(1 - sT)P(s)}$$

The characteristic equation is

$$\begin{aligned}
1 + K(1 - sT)P(s) &= 0 \\
1 + K(1 - sT)\frac{\frac{B_t}{RJ}}{s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right)} &= 0 \\
s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right) + K(1 - sT)\frac{B_t}{RJ} &= 0 \\
s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ}\right) + K\frac{B_t}{RJ} - KsT &= 0 \\
s^2 + s\left(\frac{c}{J} + \frac{B_b B_t}{RJ} - KT\right) + K\frac{B_t}{RJ} &= 0
\end{aligned}$$

The roots of this equation (i.e. the poles of the closed loop) now can be found as

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(\frac{c}{J} + \frac{B_b B_t}{RJ} - KT\right)}{2} \pm \frac{\sqrt{\left(\frac{c}{J} + \frac{B_b B_t}{RJ} - KT\right)^2 - 4K\frac{B_t}{RJ}}}{2}$$

Now we clearly see the effect of the delay of the closed loop poles.

We see that the real part of the pole can occur at the positive side of the s plane, and this will happen when $\frac{c}{J} + \frac{B_b B_t}{RJ} - KT < 0$ or when $KT > \frac{c}{J} + \frac{B_b B_t}{RJ}$

hence we see that as K is increased, the closed loop pole will move to the right until it will cross the imaginary axes making the system unstable. In addition, for a fixed gain K, as T is increased the system can become stable. An increase in T implies that the sampling frequency becomes smaller, since $f = \frac{1}{T}$.

This is what we are asked to show.