MAE 106 Laboratory Exercise #6
Vibration II: System with Two Masses

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REQUIRED PARTS

QTY	PARTS
1	50kΩ Potentiometer

EQUIPMENT
BNC to Alligator Clip Breakout
BNC Cable
Scope Probe
Oscilloscope
Breadboard
Vibrating beam experiment fixture
Accelerometer
Accelerometer Amplifier
24V DC Power Supply
Strobe Light
Spring

1 Introduction

The purpose of this laboratory exercise is to observe the free and forced dynamic response of a system of two, vibrating, cantilever beams connected by a spring. This experiment uses the same apparatus as the first vibration laboratory with a spring connecting the two vibrating masses. Our goal is to demonstrate that, although the mathematics that represent the vibrations of the system may appear complicated (4^{th} order system, as opposed to the 1^{st} and 2^{nd} order systems that we have been examining), the non-intuitive, predicted results are very real and extremely useful for machine design. You will observe that the system has two dominant modes of vibration, and that there is a frequency at which forced vibrations of one mass can be completely eliminated. Many mechanical systems, including your washing machine, use the type of vibration isolation demonstrated in this experiment. Mathematically, the vibration isolation frequency corresponds to a zero in the transfer function.

2 What are the Theoretical Resonant and Vibration Isolation Modes for the Beam System?

<u>You can do this part before coming to lab</u>

Q1 Derive the transfer function for the beam system, in matrix format. Note that this system has two inputs (the external forces applied to each beam), and two outputs (the position of each beam).

Q2 Derive the equations for the 2 resonant frequencies, ω_1 and ω_2. Show your work clearly. Then plug in numbers to obtain their values (in Hz). Assume that the second mass weighs 2.5 lb. and the spring constant of the center spring is 14 lb./in. You must also measure the beam lengths for the calculations.

Q3 Derive the vibration isolation frequency, ω_o (in Hz), corresponding the zero in the transfer function. Show your work clearly.

3 What are the Experimentally Measured Resonant and Vibration Isolation Modes for the Beam System?

You will now use a motor attached to one of the beams to create a sinusoidal forcing function. You will vary the frequency of the forcing function much as you did in the last vibration experiment. You will try to find the three frequencies of interest: the resonant frequency corresponding to the first mode, the frequency at which vibration isolation occurs, and the resonant frequency corresponding to the second mode of vibration.

Q4 Calibrate the accelerometer. Set the zero voltage adjustment on the instrumentation amplifier to give zero volts on the scope. Then rotate the entire apparatus on its side so that the accelerometer reads the acceleration of gravity (1 g). Report the accelerometer output voltage corresponding to 1 g. Explain the purpose of doing this task.

Q5 Using the strobe light and an input voltage of 2 volts from the function generator (with zero amplitude), determine the actual rpm of the motor. Be sure to hold the beam so that it does not vibrate much. Repeat the measurement for an input of 4 volts. Based on these measurements, what is your estimate of the average gain? State your answer in hertz/volt. What is the purpose of doing this task?

Q6 Measure the resonance corresponding to the first mode by slowly increasing the speed of the motor from rest. Report the motor voltage at this frequency and the corresponding frequency (in hertz).

Q7 Determine and report the vibration isolation frequency (in Hz) by increasing the frequency of the forcing function beyond the first resonance. Describe the behavior of the system at this frequency and estimate the frequency of at which this behavior occurs.

Q8 While leaving the system operating at the vibration isolation frequency, what happens when you hold the second mass from vibrating with your hand? Explain why this behavior occurs.

Q9 Determine and report (in Hz) the second resonance by slowly increasing the motor speed.

Q10 There is a third mode of vibration not predicted by the mathematics. Find it experimentally and describe it in words.

Q11 In a table, report the two resonant frequencies and vibration isolation frequency as derived in your theoretical analysis and as measured from the forced response. Explain what might cause any observed differences in the experimental and theoretical values.

PRACTICAL EXAM: Demonstrate to the TA the first two resonant modes, the vibration isolation phenomenon, and the third mode of vibration not predicted by the mathematics.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words and to type the write-up!!
- include your name and laboratory time on the write-up

1. A more realistic model of the system has some damping represented for the beams. Suppose that the two dampers are placed in parallel to the springs that represent the beams k_1 and k_3 in the notes. Denote them as c_1 and c_3 respectively.

a. Draw a schematic of the system and the necessary free-body diagrams. Derive the modified equations of motion for the system.

b. Determine the transfer function $\frac{X_1(s)}{F(s)}$, where x_1 is the displacement of the forced mass.

c. Show that it is no longer possible to get perfect vibration isolation of x_1, but that if the damping is small as in our case, the amplitude of vibration of x_1 can be made small.

2. A planar model of an automotive suspension is shown in Figure 1. The position of the center of mass, x, is zero when the springs are not deformed. The pitch motion, θ, is positive in the clockwise direction and is zero in the undeformed spring position. You can forget about gravity in the following questions since its effect only adds constant offset displacements to the equilibrium positions.

a. Derive the equations of motion for this system, assuming that the pitch motion is small.

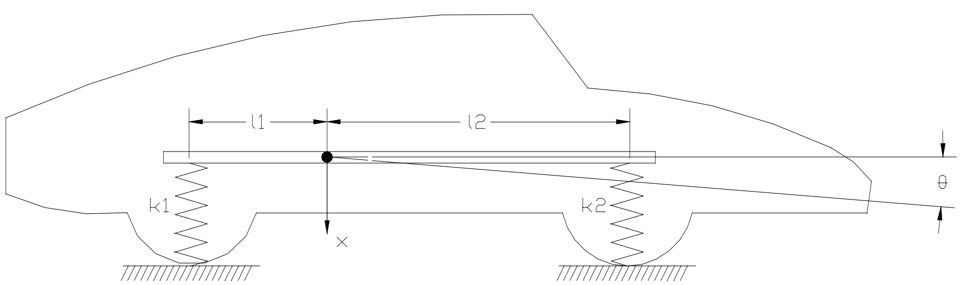

Figure 1 A planar model of an automotive

Denote the mass of the car as m and the inertia about the center of mass as J. (Hint: Assume x and θ are both positive and draw a free-body diagram.)

b. Set up the eigenvalue problem for this system, i.e. write the equations in the form

$$\left[M^{-1}K - \omega^2 I\right]\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

c. Determine the natural frequencies and their corresponding mode shapes for a 2500 lb. car with

$k_1 = k_2 = 4000$ lb/ft

$l_1 = 4$ ft.

$l_2 = 5$ ft.

inertia about the center of mass, $J = mr^2$ (m is the mass of the car)

radius of gyration, $r = 3$ ft.