MAE 106 Laboratory Exercise #6 Solution
Vibration II: System with Two Masses

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Q1

$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$ where f_1 = forcing function, f_2 = 0 for our system

$M\ddot{\overline{x}} + K\overline{x} = f$ in matrix/vector notation; taking Laplace transform:

$(Ms^2 + K)X = F$

$X = (Ms^2 + K)^{-1}F$ the transfer function relates the input F to the output X

$X = H(s)F \quad H(s) = (Ms^2 + K)^{-1}$

$$H(s) = \frac{1}{m_1m_2s^4 + (m_2(k_1+k_2) + m_1(k_2+k_3))s^2 + k_1k_2 + k_1k_3 + k_2k_3}\begin{bmatrix} m_2s^2 + k_2 + k_3 & k_2 \\ k_2 & m_1s^2 + k_1 + k_2 \end{bmatrix}$$

Q2 Resonant frequencies occur where H(jω) blows up (i.e. denominator ⇒ 0).
Note that the denominator of H(jω) is of the form:

$a\omega^4 + b\omega^2 + c$

with

$a = m_1m_2 \quad b = -(m_2(k_1+k_2) + m_1(k_2+k_3)) \quad c = k_1k_2 + k_1k_3 + k_2k_3$

So we can use the quadratic equation to find where the denominator ⇒ 0 :

$$\omega_1^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \omega_2^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Q3 The vibration isolation happens when the transfer function relating f_1 to position x_1 has a zero. I.e. H_{11}(jω)=0 where $H_{11}(j\omega) = -m_2\omega^2 + k_2 + k_3 = 0 \Rightarrow \omega_o = \sqrt{\frac{k_2+k_3}{m_2}}$

Q4 The purpose of calibrating the accelerometer is so that you know what voltage corresponds to what acceleration.

Q5 The purpose of this task is to determine what motor voltage corresponds to what forcing function frequency.

Q6 At the first resonant frequency, the beams' vibrations should get large, and the beams should move in phase with each other.

Q7 At the isolation frequency, the beam with the motor on it will stop moving while the other beam moves a lot. Imagine the beam with the motor on it is the casing for your washing machine.

Q8 The beam with the motor on it starts moving again because you are removing the vibration isolation.

Q9 At the second resonant frequency, the beams' vibration should again get large, and the beams should move out of phase with each other.

Q10 The spring has mass so you can get the spring's mass to start resonating at higher frequencies.

Q11 Differences in the experimental and theoretical values could be caused by errors in calibrating the motor or accelerometer, by errors in estimating the beam parameters, and/or by no including some dynamics, such as friction and damping, in your model.