Solution

MAE 106 Laboratory Exercise #6 Quiz
Modes of Vibration of a System with Two Masses

In this laboratory exercise you observed the free and forced dynamic response of a system of two, vibrating, cantilever beams connected by a spring. You observed that the system had two dominant modes of vibration, and that there was a frequency at which forced vibrations of one mass could be completely eliminated. This vibration isolation phenomenon is used in washing machines and other vibrating machinery.

15 pts 1. The vibration isolation frequency corresponded to a zero in the transfer function.

15 2. The resonant frequencies corresponded to poles in the transfer function.

15 3. At the first resonant frequency, the beams oscillated in phase or out of phase with each other (circle one).

15 4. How would quadrupling the stiffness of the center spring alter the vibration isolation frequency, assuming the stiffness of the beams is relatively small?

$$\omega_0 = \sqrt{\frac{k_2+k_3}{m_2}} = \sqrt{\frac{4k_2+k_3}{m_2}} \approx \sqrt{\frac{4k_2}{m_2}} = 2\sqrt{\frac{k_2}{m_2}}$$

$\boxed{\text{It would double the isolation frequency}}$

15 5. When you included some damping in the model of the system, the magnitude of the numerator of the transfer function was $\sqrt{(k_2+k_3-m_2\omega^2)^2+(c_2\omega)^2}$, where c_2 is the damping coefficient of the unforced beam. Using this information, explain why you could never get perfect vibration isolation with a damped beam.

Because $(c_2\omega)^2$ is always positive, and thus the magnitude of the numerator never equals zero.

25 6. Derive the transfer function for the beam system, in matrix format. Assume that this system has two inputs (the external forces applied to each beam), and two outputs (the position of each beam). You do not need to perform the matrix inversion, but define what each matrix is before the inversion.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Assume damping $= 0$

$$M\ \underline{\ddot{x}} + K\ \underline{x} = \underline{F}$$

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$\downarrow$ Laplace

$$(Ms^2+K)\underline{x} = \underline{F} \rightarrow \underline{x} = \underbrace{(Ms^2+K)^{-1}}_{\text{Transfer Function}}\underline{F}$$