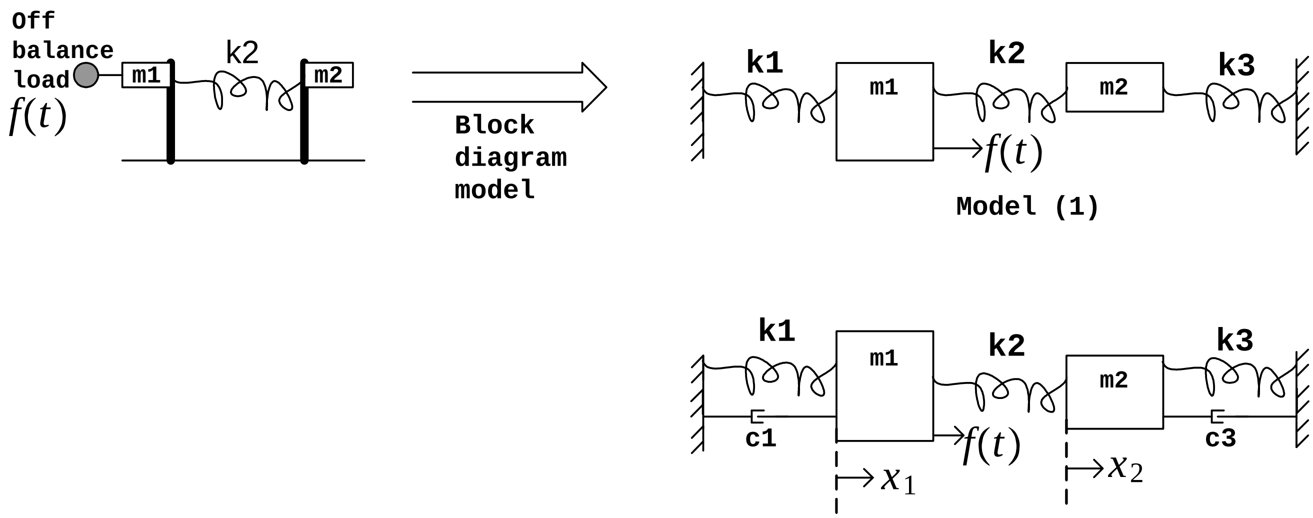
LAB #6 report. MAE 106. UCI. Winter 2005

Nasser Abbasi, LAB time: Thursday 2/17/2005 6 PM

February 23, 2005

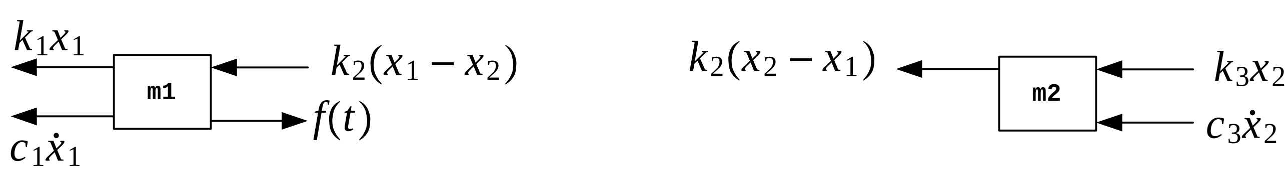
1 Answer 1.

1.1 part(a)



Model (2) more realistic

Free diagram for model (2) is the following (assuming m1 is moving to right faster than m2)



Now derive equations. Take right to be positive.
For m1:

$$\begin{aligned} F &= ma \\ -k_1x_1 - c_1\dot{x}_1 - k_2(x_1 - x_2) + f(t) &= m_1\ddot{x}_1 \\ m_1\ddot{x}_1 + \; k_1x_1 + c_1\dot{x}_1 + k_2(x_1 - x_2) &= f(t) \\ m_1\ddot{x}_1 + \; k_1x_1 + c_1\dot{x}_1 + k_2x_1 - k_2x_2 &= f(t) \\ m_1\ddot{x}_1 + \; x_1(k_1 + k_2) + c_1\dot{x}_1 - k_2x_2 &= f(t) \end{aligned}$$

For m2:

$$\begin{aligned} F &= ma \\ -k_2(x_2 - x_1) - k_3x_2 - c_3\dot{x}_2 &= m_2\ddot{x}_2 \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3x_2 + c_3\dot{x}_2 &= 0 \\ m_2\ddot{x}_2 + x_2(k_2 + k_3) - k_2x_1 \; + c_3\dot{x}_2 &= 0 \end{aligned}$$

1.2 part(b)

determine transfer function $\frac{X_1(s)}{F(s)}$

Write the dynamic equations in matrix form, we get

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_3 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}$$

Above can be written as

$$M\ddot{X} + C\dot{X} + KX = F$$

Hence , taking laplace transform we get

$$\begin{aligned} Ms^2X(s) + CsX(s) + KX(s) &= F(s) \\ X(s)\left[Ms^2 + Cs + K\right] &= F(s) \\ X(s) &= \left(Ms^2 + Cs + K\right)^{-1} F(s) \end{aligned}$$

Now, $Ms^2 = \begin{bmatrix} m_1 s^2 & 0 \\ 0 & m_2 s^2 \end{bmatrix}$

$Cs = \begin{bmatrix} c_1 s & 0 \\ 0 & c_3 s \end{bmatrix}$

$K = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}$

Hence

$$\begin{aligned} \left(Ms^2 + Cs + K\right)^{-1} &= \left(\begin{bmatrix} m_1 s^2 & 0 \\ 0 & m_2 s^2 \end{bmatrix} + \begin{bmatrix} c_1 s & 0 \\ 0 & c_3 s \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix} \right)^{-1} \\ &= \begin{bmatrix} m_1 s^2 + c_1 s + k_1 + k_2 & -k_2 \\ -k_2 & m_2 s^2 + c_3 s + k_2 + k_3 \end{bmatrix}^{-1} \end{aligned}$$

Now $A^{-1} = \frac{adj(A)}{\det(A)}$

But for the above,

$$\begin{aligned} \det(A) &= \left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - \left(-k_2 \times (-k_2)\right) \\ &= \left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - k_2^2 \end{aligned}$$

$$adj(A) = \begin{bmatrix} m_2 s^2 + c_3 s + k_2 + k_3 & k_2 \\ k_2 & m_1 s^2 + c_1 s + k_1 + k_2 \end{bmatrix}$$

Hence

$$\begin{aligned} \frac{X(s)}{F(s)} &= \left(Ms^2 + Cs + K\right)^{-1} \\ &= \frac{\begin{bmatrix} m_2 s^2 + c_3 s + k_2 + k_3 & k_2 \\ k_2 & m_1 s^2 + c_1 s + k_1 + k_2 \end{bmatrix}}{\left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - k_2^2} \end{aligned}$$

i.e.

$$\frac{X_{1(s)}}{F(s)} = \frac{m_2 s^2 + c_3 s + k_2 + k_3}{\left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - k_2^2}$$

and

$$\frac{X_{2(s)}}{F(s)} = \frac{k_2}{\left(m_1 s^2 + c_1 s + k_1 + k_2\right)\left(m_2 s^2 + c_3 s + k_2 + k_3\right) - k_2^2}$$

1.3 part(c)

Let $s = j\omega$ hence

$$\frac{X_{1(s)}}{F(s)} = \frac{-m_2\omega^2 + jc_3\omega + k_2 + k_3}{(-m_1\omega^2 + jc_1\omega + k_1 + k_2)(-m_2\omega^2 + jc_3\omega + k_2 + k_3) - k_2^2}$$

x_1 will not move when

$$\left|\frac{X_{1(s)}}{F(s)}\right| = 0 \Rightarrow \left|-m_2\omega^2 + jc_3\omega + k_2 + k_3\right| = 0$$

but $\left|-m_2\omega^2 + jc_3\omega + k_2 + k_3\right| = 0$ implies $\sqrt{(-m_2\omega^2 + k_2 + k_3)^2 + (c_3\omega)^2} = 0$. i.e. $(-m_2\omega^2 + k_2 + k_3)^2 + (c_3\omega)^2 = 0$. But this is the sum of 2 positive quantities. So it is only possible to sum to zero only when each quantity itself is zero. i.e.

$$c_3\omega = 0$$

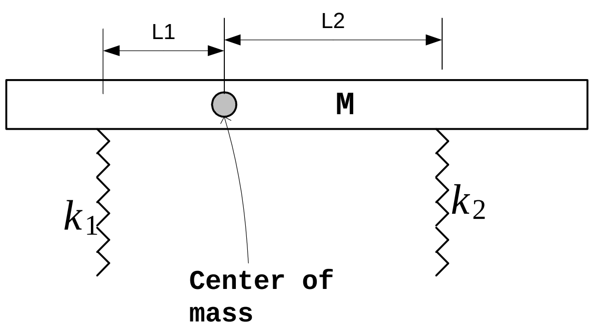
But for non zero ω this means that $c_3 = 0$. But c_3 (the samping) is not zero, since we do have damping in the systems, hence it is not possible that $\left|\frac{X_{1(s)}}{F(s)}\right| = 0$. In otherwords, there will not be an isolation fequency, and x_1 will always be non-zero.

But if c_3 is very small, then $c_3\omega = 0$ and in this case $\left|\frac{X_{1(s)}}{F(s)}\right| = 0$ when $-m_2\omega^2 + k_2 + k_3 = 0$ or when $\omega = \sqrt{\frac{k_2+k_3}{m_2}}$

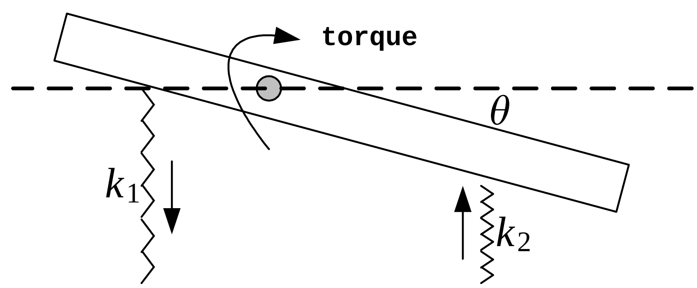
2 Answer 2.

2.1 part(a)

Need to derive a mathematical model. First step is to make a block diagram as follows.

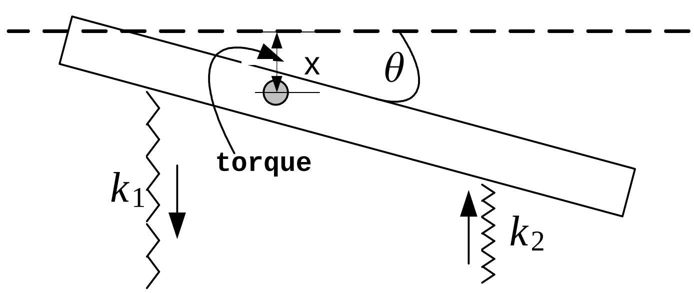


Block diagram of system with no movment



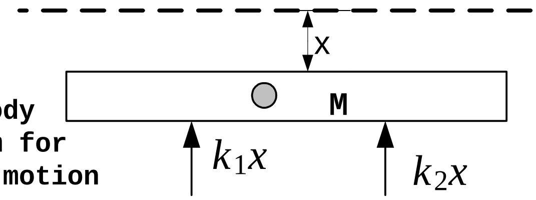
Block diagram of system with pitch motion only

Block diagram of system with pitch and vertical motion

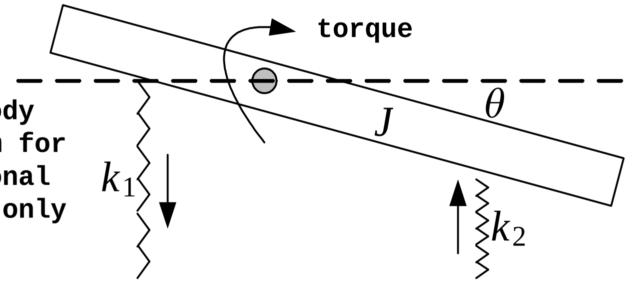


There are 2 motions. One rotational about the center of mass, and one translation, up and down.
Free body diagrams are

Free body diagram for linear motion only



Free body diagram for rotational motion only



Now the equation of motion for the rotational motion is

$$\tau = J\ddot{\theta}$$

But $\tau = k_1 L_1 \sin\theta - k_2 L_2 \sin\theta$
Hence we get for small θ, using $\sin\theta \approx \theta$

$$\begin{aligned} k_1 L_1\ \theta - k_2 L_2\ \theta &= J\ddot{\theta} \\ \theta\left(k_1 L_1 - k_2 L_2\right) &= J\ddot{\theta} \\ J\ddot{\theta} + \theta\left(k_2 L_2 - k_1 L_1\right) &= 0 \end{aligned}$$

For the translation motion, $F = ma$
hence

$$\begin{aligned} -k_1 x - k_2 x &= m\ddot{x} \\ x\left(-k_1 - k_2\right) &= m\ddot{x} \\ m\ddot{x} + x\left(k_1 + k_2\right) &= 0 \end{aligned}$$

2.2 part (b)

Write the above in matrix form, we get

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_2 + k_1 & 0 \\ 0 & k_2 L_2 - k_1 L_1 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Take laplace transform we get

Let $M = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}$

$A = \begin{bmatrix} x \\ \theta \end{bmatrix}$

$K = \begin{bmatrix} k_2 + k_1 & 0 \\ 0 & k_2 L_2 - k_1 L_1 \end{bmatrix}$

Hence above matrix equation can be written as
$M\ddot{A} + KA = 0$
Take laplace transform, we get
$Ms^2 A + KA = 0$
$\left[Ms^2 - IK\right] A = 0$ where I is the 2×2 identity matrix.
let $s = j\omega$ we get
$\left[-\omega^2 M - IK\right] A = 0$
multiply both side by M^{-1} we get
$\left[-\omega^2 I - KM^{-1}\right] A = 0$
i.e.

$\left[-\omega^2 I - KM^{-1}\right] \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Which is what we are required to show.

2.3 Part (c)

$k_1 = k_2 = 4000lb/ft$

$L_1 = 4ft$

$L_2 = 5ft$

$m = 2500lb$

$J = mr^2 = 25000 \times 3^2 = 2.25 \times 10^5$

$\omega_0 = \sqrt{\frac{K}{M}} = \sqrt{KM^{-1}}$

$$M^{-1} = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}^{-1} = \begin{bmatrix} 2500 & 0 \\ 0 & 2.25 \times 10^5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{2.25\times10^5}{2500\times2.25\times10^5} & 0 \\ 0 & \frac{2500}{2500\times2.25\times10^5} \end{bmatrix} = \begin{bmatrix} 0.0004 & 0 \\ 0 & 4444 \times 10^{-6} \end{bmatrix}$$

$$K = \begin{bmatrix} k_2 + k_1 & 0 \\ 0 & k_2L_2 - k_1L_1 \end{bmatrix} = \begin{bmatrix} 8000 & 0 \\ 0 & 4000 \times 5 - 4000 \times 4 \end{bmatrix} = \begin{bmatrix} 8000 & 0 \\ 0 & 4000.0 \end{bmatrix}$$

Hence $\omega_0 = \sqrt{\begin{bmatrix} 8000 & 0 \\ 0 & 4000.0 \end{bmatrix} \begin{bmatrix} 0.0004 & 0 \\ 0 & 4444 \times 10^{-6} \end{bmatrix}} = \sqrt{\begin{bmatrix} 3.2 & 0 \\ 0 & 17.776 \end{bmatrix}} = \begin{bmatrix} 1.7889 & 0 \\ 0 & 4.2162 \end{bmatrix}$

Hence the natural frequency for the linear (translation) motion is $\boxed{1.7889 \text{ rad/sec}}$, and for the rotational motion it is $\boxed{4.2162 \text{ rad/sec}}$.