MAE 106 Laboratory Exercise #5
PD Control of Motor Position

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REQUIRED PARTS:

Qty	Parts	Equipment
2	1kΩ resistor, ¼ W (brown/black/red)	Breadboard
2	10kΩ resistor, ¼ W (brown/black/orange)	Oscilloscope
2	100kΩ resistor, ¼ W (brown/black/yellow)	Function Generator
1	LM 324 quad op amp chip	Motor-Amp-Tach Console
4	1μF capacitors	Position-sensing "pot"
1	BNC cable	IC puller
1	breakout (BNC to alligator clips)	wrist grounding strap
2	banana-to-banana cable (1 black, 1 red)	multimeter
2	banana-to-alligator clip cable (1 black, 1 red)	scope probe
var	wire, 22AWG	
1	∅1/4" shaft coupling (c.1/2"lg)	

1 Introduction

In this lab you will build a control system to make a motor shaft move to a position that you command. Controlling motor position is a common goal in automation (e.g. multi-joint robot arms, radars, numerically controlled milling machines, manufacturing systems). In addition, you will need a position controller for your final project.

The controller that you will build is called a "Proportional Plus Derivative (PD) Position Feedback System," and is the most common controller found in industry. The PD control law is:

$$\tau = -K_p(\theta - \theta_d) - K_d\dot{\theta} \qquad (1)$$

Where θ = actual motor angular position
θ_d = desired motor angular position
$\dot{\theta}$ = actual motor angular velocity
K_p = position error gain
K_d = derivative gain
τ = desired motor torque

Note that the controller has two terms – one proportional to the position error (the "P" part), and one proportional to the derivative of position (i.e. velocity, the "D" part). Thus, it is called a "PD" controller.

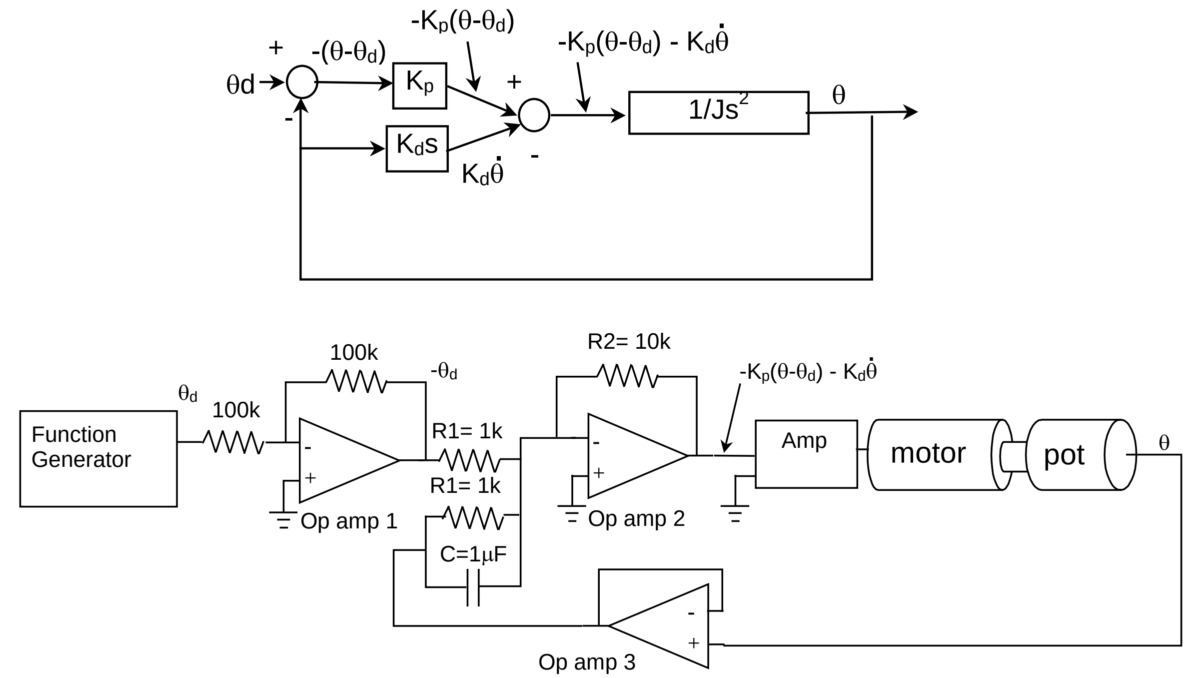

Figure 1 – PD Motor Position Control System (Block Diagram and Circuit)

Figure 1 shows the block diagram and op-amp circuit that you will implement to make the PD control law for the motor. J is the inertia of the motor shaft.

The lab has the following four parts. Y<u>ou can do Parts 1 and 2 before coming to lab.</u>

Part 1: What is the theoretical behavior of the controlled system?
The key point to understand here is that the controlled system obeys the same differential equation as a mass-spring-damper system. Thus, the controlled system acts dynamically like a mass-spring-damper system. The control gains K_p and K_d determine the equivalent stiffness and damping of the system. The desired angular position of the motor (θ_d) is equivalent to the rest length of the spring.

Part 2: How can a circuit implement the control law?
Op-amp circuits (adder, gain, inverter derivative circuits) can be used to implement the control law. The resistors and capacitors set the control gains K_p and K_d.

Part 3: What is the step response of the actual system?
One way to characterize the system behavior is to measure how it performs when it is commanded to move rapidly from one position to another (i.e. to follow a step function input). You will find that the motor will overshoot and oscillate if the damping is too small.

Part 4: What is the frequency response of the actual system?
Another common way to characterize the system behavior is to measure how it performs when it is commanded to follow a sinusoidal position. You will find that the controller acts like a low pass filter. It tracks low input frequencies well, and high frequencies poorly. Also, if the controller has low enough damping, it will resonate just like the spring-mass-damper system you experimented with in Lab 4.

2 What is the theoretical behavior of the controlled system?

In this section, you will derive the theoretical behavior the PD position controlled motor. In the time domain, the theoretical behavior is described by a differential equation. In the frequency domain, the theoretical behavior is described by the frequency response.

Q1 Derive the dynamical equation that describes how θ evolves with time when the controller is attached to the motor. Assume θ_d is the input.

Q2 Derive the differential equation for a spring-mass-damper system (assume force is the input and position is the output). The differential equation for Q1 should be similar to the equation for Q2. This means that the PD position control system has the same dynamics as a spring-mass-damper system; i.e. it follows the same equations of motion. Thus, you can use your intuition about how the spring-mass-damper system works to design the PD controller. Explain what the mass (m), spring (k), damper (c) in the mechanical system correspond to in the PD system.

Q3 Derive the closed-loop transfer function, G(s), for the controlled system (the input is θ_d, the output is θ). Use either block diagram algebra (applied to the block diagram from Figure 1) or take the Laplace Transform of the differential equation that you derived in Q1.

Q4 Express the damping ratio and natural frequency of the system in terms of the control gains and motor inertia. The damping ratio is important because it determines whether the system oscillates. The natural frequency determines the frequency at which it oscillates.

P1 Plot the predicted response of the system to a step change in θ_d from 0 to 1 radians, for damping ratios of 0.1, 1.0, and 2.0.

P2 Plot the predicted frequency response (both scaling and phase shift) for damping ratios of 0.1, 1.0, and 2.0. Do this on a Bode plot by plotting {20log(output amplitude/input amplitude)} vs. {input frequency on a log scale}, and {phase shift} vs. {input frequency on a log scale}.

3 How can a circuit implement the control law?

To implement the PD control law, you need to build the circuit shown in Figure 1.

Q5 By applying the op-amp golden rules, show that the input to the motor amplifier is:

$$Vout = -\frac{R_2}{R_1}(\theta - \theta_d) - R_2 C\dot{\theta}$$

The derivation will be easier if you substitute the impedance 1/sC for the capacitor then treat it as a resistor in the frequency domain, then transform back to the time domain.

Q6 Compare this equation with the control law of equation (1). What are K_P and K_d in terms of the electronic components (resistor and capacitor values)? How would you increase the damping of the system?.

Q7 Briefly describe the specific purpose for each of the op-amps in Figure 1.

Part 3 What is the step response of the actual system?

Construct the circuit in Figure 1. *Important: wire your circuit neatly! A neat circuit requires little extra time to wire, and it's easier to debug. Circuits often take much more time to debug than they do to initially wire!* Make sure to hook up the potentiometer correctly! The wiper of the pot should **not** be connected to the power supply!

If your circuit works correctly, the motor position should follow whatever input signal you provide with the function generator (square wave, sinusoid, constant voltage…).

IMPORTANT: Sometimes the circuit will not work and the motor will run uncontrollably at a very high speed. This wrecks the pots. If this happens, turn the motor off right away by turning of the DC supply to the motor. First, try to fix the instability problem by reversing the polarity across the sensing pot (you may have positive feedback instead of negative feedback). If this doesn't work, debug your circuit. Do not try to debug your circuit with the motor running! Debug your circuit systematically.

Here are some debugging hints:
- Compare your wiring diagram to your circuit to make sure all of the connections are correct.
- Make sure you don't have any loose connections.
- Verify that your output pot is working properly by connecting the scope to the wiper and moving the motor shaft by hand. The scope trace should move up and down.
- Verify that op-amp 1 is inverting and op-amp 3 is following.
- Verify that the output of op-amp 2 changes as you adjust θ_d with θ constant. You can adjust θ_d using the function generator or another pot.

Q8 Provide a step-input by using the function generator (4V peak-peak, 1 Hz square wave). Is the system underdamped or overdamped? Does the observed response agree with the theoretical one? Why is it different? What is the frequency at which is oscillates (the damped natural frequency, ω_{damped}).

PRACTICAL EXAM: Demonstrate to the TA that your motor is following the step input.

P3 Suppose you didn't want your motor to oscillate so much. **This is an important issue!** PD controllers are used in many applications such as NC milling machines, plotters, etc. You usually want your motor to go to a desired value quickly and accurately without oscillating! Which variable would you change in your differential equation for the PD system to increase damping? Increase the total capacitance to 2μF by adding another capacitor (recall that capacitors add in parallel). Observe the response to a step input. Repeat this for a total capacitance of 3 μF and 4 μF. Record the step response using the LabJack for C = 1, 2, 3, and 4 μF.

P4 In a mechanical system, if you wanted to the system to respond more quickly, you would increase the natural frequency (ω_n) by picking a stiffer spring (higher k). Double ω_n for the PD system by changing the appropriate resistor value. Keep C = 4 μF. Record the step response of the system using the LabJack, and plot it on the same plot as P3.

Part 4: What is the frequency response of the actual system?

The goal of this last part of the lab is to characterize the frequency response of the system. In particular, you will explore how well the system tracks the desired input position when the input is a sinusoid, across a range of frequencies. Remember, you can view linear systems such as this one as "filters". The PD controller acts like a low-pass filter, although it has a resonant peak if the damping is not great enough.

Q9 Change R1 to 1 KΩ, R2 to 10 KΩ, and C = 1 μF. Now input a 1.0V amplitude sine wave. Start at a low frequency. Keep the voltage scale on the scope the same for both input and output. What is the output amplitude and phase shift at 1, 2, 4, 8, 16, 32 Hz? Does the system have a resonant frequency? The amplitude should increase dramatically at this point. What is the resonant frequency and the output amplitude at resonance? Do you notice any high frequency oscillations in your output signal? What do you think might be causing those? **Note:** Don't let the motor oscillate for a long time at resonance.

P5 Make a Bode plot of the frequency response of the system. Plot {20log(output amplitude/input amplitude)} vs. {input frequency on a log scale}, and {phase shift} vs. {input frequency on a log scale}.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words and to type the write-up!!
- include your name and laboratory time on the write-up
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.

Page limit = 2 pages, including graphs

1. A controller that performs a little better than the PD controller used in this lab is the following:

$$\tau = -K_p(\theta - \theta_d) - K_d(\dot{\theta} - \dot{\theta}_d)$$

   a. Derive the closed-loop transfer function for this controller.
   b. Provide a reason why this controller performs better in tracking a changing desired position input.
2. Step Response: Turn in the plots for P1, P3 and P4
3. Frequency Response: Turn in the plot for P2 and P5.