MAE 106 Laboratory Exercise #5 Solution
PD Control of Motor Position

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Q1 Dynamics of motor and shaft: $J\ddot{\theta} = \tau$

Dynamics of controller system: $\tau = J\ddot{\theta} = -K_p(\theta - \theta_d) - K_d\dot{\theta}$

Re-writing to make input and output clear: $J\ddot{\theta} + K_d\dot{\theta} + K_p\theta = K_p\theta_d$

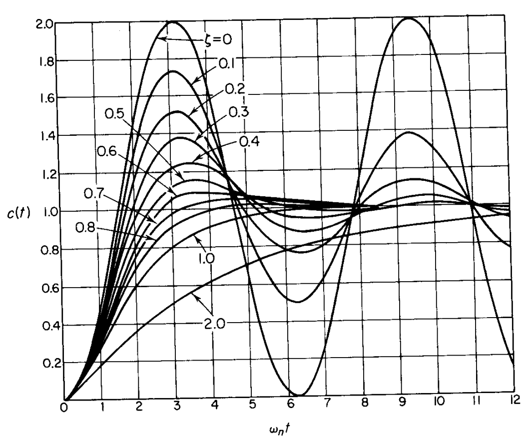
Q2 $M\ddot{\theta} + B\dot{\theta} + K\theta = F$

Mass = motor inertia (m = J), spring = Proportional control term (k = K_p) damper = Derivative control term (c = K_d)

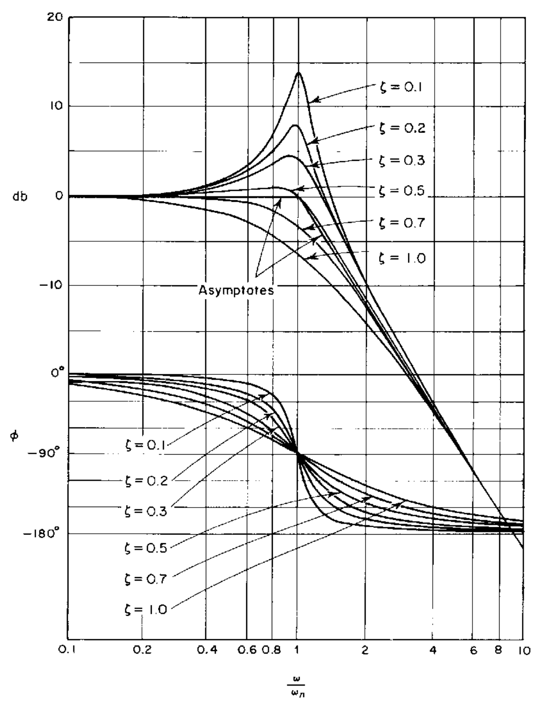
Q3 $$G(s) = \frac{K_p}{Js^2 + K_d s + K_p}$$

Q4 $$\omega_n = \sqrt{\frac{K_p}{J}} \qquad \zeta = \frac{K_v}{2\sqrt{K_p J}}$$

P1



P2



Q5 $\frac{-\theta_d}{R_1} + \frac{\theta}{Z} = \frac{-V_{out}}{R_2}$ $\qquad Z = \frac{R_1 \frac{1}{sc}}{R_1 + \frac{1}{sc}} = \frac{R_1}{1 + R_1 cs}$

$$Vout = -\frac{R_2}{R_1}(\theta - \theta_d) - R_2 Cs\theta$$

Converting back to time domain:

$$Vout = -\frac{R_2}{R_1}(\theta - \theta_d) - R_2 C\dot{\theta}$$

Q6 $K_P = R_2/R_1$ $K_d = R_2C$; Increase damping by increasing R_2 or C

Q7 Op amp 1 creates $-\theta_d$ for use in the control law
Op amp 2 implements the control law equation as in Q5
Op amp 3 is a buffer so that the motor potentiometer is not loaded by the rest of the circuit

Q8 Underdamped: $f_{damped} \approx 24$ Hz $\qquad \omega_{damped} = 2\pi f_{damped}$

P3 As you add capacitors, the damping increases and the oscillations decreases. With 2 C's you should have about 2 oscillations, 3 C' gives 1.5 oscillations, 4C's gives about 1 oscillation.

P4 Rise time and peak time should be much faster now.

Q9 Resonant frequency should be around 24 Hz, output amplitude should be about 1.5 times input amplitude at resonance.
Possible cause of the high frequency oscillations: compliance in the coupling between the potentiometer and motor.

P5 See plot on P1.