MAE 106 Laboratory Exercise #4

Vibration I: Lightly Damped Second Order Systems

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Required Parts:

QTY	PART	
1	50kΩ Potentiometer	Oscilloscope
		Breadboard
	EQUIPMENT	Vibrating beam experiment fixture
	BNC to Alligator Clip Breakout	Accelerometer
	BNC Cable	Accelerometer Amplifier
	Scope Probe	24V DC Power Supply
	Strobe Light	

1 Introduction

In this laboratory exercise you will look at the dynamic response of a cantilevered beam that supports a motor with an unbalanced load attached at its end. If you use your imagination, this system represents many typical problems in vibrations. For instance, a large rotating machine attached to a building floor can be analyzed in the same manner as this experiment, with the floor taking the place of the cantilevered beam. Alternatively, a building shaking during an earthquake can also be represented by the same equations, with the building acting as the beam itself.

This lab is also the first lab that deals with a second order system. In other words the differential equation that describes the system has second derivatives, and the transfer function has s^2 terms. Many mechanical systems behave as second order systems because of Newton's second law (F = ma) because acceleration is the second derivative of position. In fact, you can view a vast number of mechanical and control systems as second order linear mass, spring, damper systems. Thus, developing intuition about how second-order systems behave is very important. One key difference between second and first order systems, as you will see in this lab, is that second order systems can oscillate. First order systems cannot oscillate.

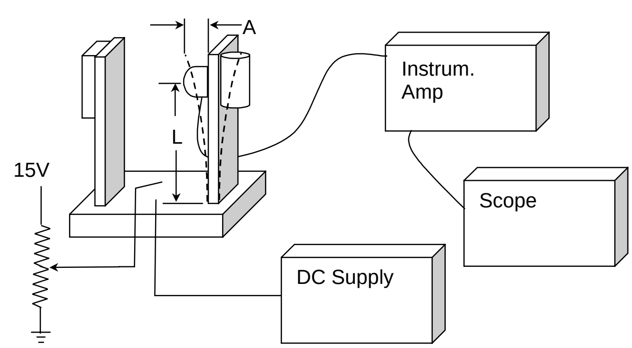


Figure 1 - Vibrating Beam Fixture: Be gentle with the beam fixture. The accelerometer can be easily damaged by impulsive forces. Also, place the beam fixture on the floor, being careful to not to pull any short wires.

2 Time Domain Analysis (Transient Response)

In this part of the lab, you will measure how the beam responds to an impulsive input. This is known as the "transient response", and more specifically, as the "impulse response" of the beam, and is a typical way to look at the time-domain response.

An accelerometer is mounted at approximately the center of mass of the vibrating load at the beam end. For all the subsequent analysis, assume that the length of the beam is from the clamped end to the center of the accelerometer. The following analysis also assumes that the *acceleration* of the beam is a good measure of its *position*. The reason for this assumption is that the equation of motion of the unforced system has the form

$$m\ddot{x} + c\dot{x} + kx = 0,$$

and if c ≈ 0, then $\ddot{x} = (-k/m)\,x$.

Q1 Compute the theoretical natural frequency of the system. The motor weighs about 2.25 lb., the beam is made of carbon steel of dimensions 0.125 in. by 2.0 in. in cross-section. You need to measure the length of your beam as the distance from the base to the center of the accelerometer. (You can do the calculations at home but be sure to measure the length of your beam, since they are all different!)

Connect the accelerometer to the instrumentation amplifier. Attach the amplifier output to the oscilloscope and set the amplifier gain to x5.

Q2 You need to calibrate the accelerometer to make sense of its output. **"**Calibrating" a sensor refers to the process of measuring what voltage corresponds to what level of the measured variable. For the accelerometer, you need to know how the output voltage and acceleration correspond. You can use gravity as your known acceleration, and measure the voltage output corresponding to gravity. Set the zero voltage adjustment on the instrumentation amplifier to give zero volts on the oscilloscope. Then rotate the entire apparatus on its side so that the accelerometer reads the acceleration of gravity (1 g). Report the accelerometer output voltage corresponding to 1 g. You are now able to calculate actual acceleration by measuring the accelerometer voltage. Give an example of how you would do this. What assumptions are you making about the accelerometer and amplifier?

Q3 Twang the beam with your hand and set the oscilloscope so that a nice periodic waveform appears on the screen. Report the frequency of vibration (both in rad/sec and Hz). Use the **stop** button on the scope for this.

Q4 Estimate the damping using the logarithmic decrement method. Use the **stop** button on the scope and a slow sweep rate to obtain a good scope trace. After twanging the structure, measure the initial amplitude, the number of cycles, and the final amplitude. Repeat this process a few times to be certain of your measurement. Hint: use "Roll" (horizontal mode) and storage mode of the scope. Report you estimated value of ζ using both the exact and approximate formulas in Equation 3 of the notes.

P1 Using the LabJack, record the impulse response of the beam. You will turn this plot in.

3 Frequency Domain Analysis (Forced Response)

In this part of the lab, you will determine how the beam responds when you apply sinusoidal forces to it at different frequencies. A key phenomenon that you will observe is resonance. Resonance is the increase in amplitude of oscillation of an electric or mechanical system exposed to a periodic force whose frequency is equal or very close to the undamped natural frequency of the system.

You will use a motor with an off-balance load to apply the sinusoidal forces to the beam. The motor is driven by a high gain, *velocity control system*. The voltage into the amplifier and controller produces an angular velocity of the motor with a gain of about 300 rpm/volt (but you need to measure this to find the exact value). So if you input zero volts (a short) to the amplifier, you should get zero rpm out, while a 2 volt input would give 600 rpm out. Use a potentiometer in a voltage divider circuit on the breadboard, or the function generator, to provide a variable voltage input to the motor velocity control system.

Q5 It is important to know the relation of the motor's velocity to input voltage accurately for the system (i.e. to calibrate the motor). Using the strobe light and an input voltage of 2 volts, determine the actual rpm of the motor. Be sure to hold the beam so that it does not vibrate much. Repeat the measurement with an input voltage of 4 volts. Based on these measurements, what is your estimate of the gain that relates voltage to velocity? State your answer in rpm/volt and in (rad/sec)/volt. Now that you've calibrated the system, you can estimate the motor velocity by measuring input voltage. Conversely, you can adjust motor velocity by adjusting the input voltage.

Q6 Try to estimate the natural frequency, ω_n, for the system by getting it to resonate. The motor angular velocity corresponding to the maximum amplitude response gives the resonant frequency ($\omega_r = \omega_n$). Try not to let the system shake too badly, i.e. do not let the system stay in resonance too long. Record the accelerometer voltage amplitude at resonance. What is the corresponding maximum acceleration? Also, estimate the amplitude of the tip beam motion in inches at resonance with a ruler as shown in Figure 1.

Q7 Increase the voltage to the motor controller so that you are exciting the system well past its resonant frequency, but not to the point where new, higher frequency resonance is occurring (you can hear other things begin to shake). Record this input voltage. As above, record the accelerometer amplitude voltage and estimate the amplitude of the beam tip motion, A_{high}, with a ruler by eye.

PRACTICAL EXAM: Demonstrate to the TA that you can drive your beam into resonance.

3 Problems to Consider at Home

You are now done with the experimental part of the lab. The rest is analytical.

Q8 It was shown in lecture that the forcing function on the mass has the form

$$f(t) = a\omega^2 \sin(\omega t).$$

Therefore, the output must be of the form

$$a\omega^2 \left|G(j\omega)\right| \sin(\omega t + \phi), \text{ (1)}$$

where
$$G(s) = \frac{w_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

When the beam is vibrating near resonance, $|G(j\omega_n)| = 1/(2\zeta)$, and the amplitude of the output wave (that you measured with a ruler) is

$$A_{res} = \frac{a\omega_n^2}{2\zeta}.$$

In Q12, below you will prove that at higher frequencies, as $\omega \to \infty$,

$$a\omega^2 |G(j\omega)| \to a\omega_n^2.$$

Therefore, the amplitude of the output wave (also measured with a ruler) is $A_{high} = a\omega_n^2$.

From these two facts, the damping ratio ζ can be estimated as

$$\zeta \approx \frac{A_{high}}{2A_{res}}. \quad (2)$$

Using this formula and the amplitudes measured by eye in Q6 and Q7, estimate the damping ratio, ζ.

Q9 Prove that the resonant frequency ω_r which is the frequency of maximum output vibration amplitude, is given by

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}, \quad \zeta \le 0.707.$$

Q10 Show that

$$|G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}.$$

Q11 Show that as $\zeta \to 0$, $\omega_r \to \omega_n$, and that

$$|G(j\omega)| \to |G(j\omega_n)| = \frac{1}{2\zeta}.$$

Q12 Show that as $\omega \to \infty$, $\omega^2 |G(j\omega)| \to \omega_n^2$.

Q13 Make a table of the estimates of the natural frequency ω_n that you computed using the three methods (Q1, Q3, and Q6) and compare them. Which do you think is the most accurate? Why?

Q14 Make a table of the estimates of the damping ratio ζ that you computed using the methods of Q4, and Q8 and compare them. Which do you think is the most accurate? Why?

Q15 Using your measured values of ζ, ω_n, and m, estimate the viscous damping constant c from the notes. Be sure to state your units.

Q16 Show that if you know the natural frequency, ω_1, for one beam of length l_1, you can estimate the natural frequency for any other beam of length l_2 with the equation

$$\omega_2^2 = \left(\frac{l_1}{l_2}\right)^3 \omega_1^2.$$

Use this formula to estimate the natural frequency of a 15-in. long the beam with the same properties. Note: To use this equation, you must assume that tip mass is constant or that the beam is massless.

WRITE-UP

- due at your next laboratory session
- each student must complete his or her own write-up
- make sure to use your own words!!
- include your name and laboratory time on the write-up
- the write-up must be type-written
- Graphs for the lab write-up must be generated using Excel or Matlab, and must include labels on the axes, voltage and time scales used on the scope, and a legend for multiple-line plots.
- Page limit = 2 pages, including graphs

1. Plot the impulse response of the beam for several cycles (Plot P1).

A schematic representation of the accelerometer used in the experiments is shown in Figure 2.

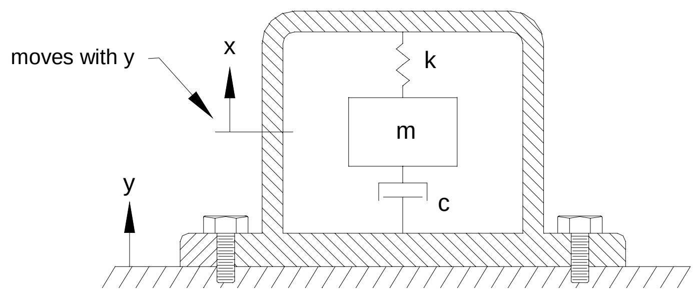
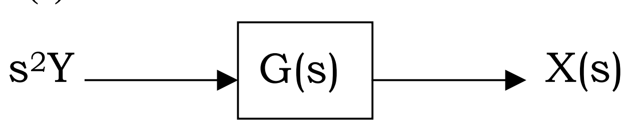


Figure 2 Schematic of accelerometer. Note the way in which the x-axis moves with the y-axis, i.e. if the entire accelerometer moves, the mass must move with it.

The variable y is the position of the housing relative to a fixed reference frame, and x is the position of the internal mass relative to the housing. The position of the internal mass with respect to a fixed reference frame is, therefore $x + y + constant$. The acceleration $\ddot{y}$ causes the internal spring to deform some distance x. The spring deformation x is measured electrically with a capacitive distance sensor, and this voltage is output to the instrumentation amplifier.

2. Derive the differential equation of motion for the accelerometer in terms of x, y, and their derivatives.
3. These acceleration measurement dynamics can be represented by a transfer function with acceleration, s^2Y as the input and spring deformation x as the output, as shown. Determine the transfer function $G(s)$.



4. The manufacturer's data sheet for the accelerometer is attached. The model we used is the (+/-)2-g nominal range unit. Determine the values of k/m and c/m for this accelerometer from the data sheet.

SETRA SYSTEMS, INC.
HIGH OUTPUT LINEAR ACCELEROMETER

Model 141

FOR VIBRATION, SHOCK, IMPACT
Ranges from: ±2g to ±600g
With External R_{cal} Calibration

Features

- **Excellent static and dynamic response**
- **Temperature-insensitive gas damping (0.7 critical)**
- **High output signal**
- **High overload capability, (2000g static)**
- **Low transverse sensitivity (0.005 g/g)**
- **Wide-range R_{cal} type calibration**
- **Easy-to-replace cable attachment**
- **Compact, lightweight**

Description

The Model 141 is a linear accelerometer that produces a high level instantaneous DC output signal proportional to sensed accelerations (ranging from static acceleration up to 3000 Hz as reported below).

Setra accelerometers are unique in their ability to withstand exceedingly high g overload without damage. The Model 141 incorporates the super-rugged Setra capacitance-type sensor and a new miniaturized electronic circuit.

Its excellent dynamic response is maintained by air damping, which varies with temperature approximately one-tenth as much as the best fluid damping.

The electrical characteristics are compatible with conventional strain-gage type signal conditioning, including the use of shunt R_{cal} over any selected range up to 100% full scale.

The stainless steel case is O-ring sealed, has a well defined base plane, is quite insensitive to mounting strain.

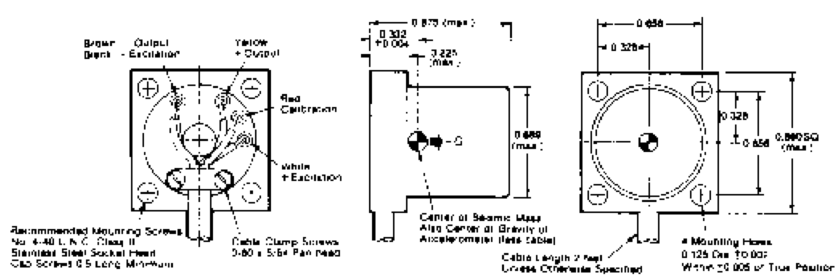
Cross axis interference is exceedingly low. The external easy-to-replace cable attachment facilitates installation and service.

Full Scale Ranges

For each of the available g ranges, the linearity is characterized by this range chart: (Non-linearity as % full range, best straight line).

Nominal Range	Non-Linearity			Natural Frequency (Nominal)	Flat Response (±3 db) 0 Hz to:
	±0.5%	±1%	±3%		
±2g	±1.5g	±2g	±2.5g	275 Hz	200 Hz
±4g	±3g	±4g	±5g	330 Hz	260 Hz
±8g	±6g	±8g	±10g	350 Hz	300 Hz
±15g	±10g	±15g	±20g	600 Hz	400 Hz
±30g	±20g	±30g	±40g	1150 Hz	700 Hz
±60g	±40g	±60g	±80g	1600 Hz	1000 Hz
±150g	±100g	±150g	±200g	2600 Hz	1600 Hz
±600g	±400g	±600g	±800g	5000 Hz	3000 Hz

Outline Drawing



Note: All dimensions are in inches.

setra

45 Nagog Park, Acton, Massachusetts 01720 / Telephone: (617) 263-1400

Model 141 Specifications

Ranges, Non-Linearity, Frequency Data. Please refer to chart on front page.

Other Accuracy Data

Hysteresis	<±0.1%
Non-Repeatability	<±0.05% Nominal range
Transverse Acceleration Response	<±0.005 g/g
Damping	Approximates second order system with 0.7 critical damping. The frequency band for all ranges is flat from static to approximately 60% of the natural frequency. Damping is gas squeeze-film, 0.7 ±0.2 of critical at 77°F. Damping ratio increases approximately 0.15%/°F.
Resolution	Infinite, limited only by output noise level.
Thermal Effects	Operating temperature −10°F to 150°F Zero shift <±0.02% Nominal Range/°F Sensitivity shift <±0.02% Nominal Range/°F Slightly higher thermal effects when 141A is operated at excitation voltage below 10VDC. **Model 141A (special order) −65°F to 220°F**
Zero G Output	<±25 mv (factory calibrated at designated excitation)
Noise Level	<±0.01% Nominal Range (RMS, in-band)
Calibration Data	Each unit is supplied with a full scale continuous plot of output vs. acceleration (centrifuge), at a designated excitation voltage. Sensitivity is reported at Nominal Range. Model 141A calibrated at 10VDC excitation. Model 141B calibrated at 24VDC excitation.

Electrical Data

Electrical Circuit	Three-terminal equivalent, common -excitation and -output signal. Circuit is capacitively isolated from case, greater than 100 megohm isolation. Power applied to output, or shorted output, will not damage unit. No reverse excitation protection. Operates at internal frequency approximately 20 MHz. Model 141B operable on regulated 28 VDC aircraft power, (recommend high voltage transient protection to prevent damage by emergency power conditions as defined in MIL-STD-704A, and voltage regulation to attain highest accuracy).
Calibration Signal (R_{cal})	Available up to 100% Nominal Range by shunting external calibration resistor from calibration lead to -signal lead.
Voltages and Currents	Two versions are available, offering your choice of units for different excitation voltages. Output is proportional to excitation voltage. Output impedance 9K ohms (nominal).

Typical performance for nominal G range:

Model	Excitation Range	At Excitation Voltage of:	Excitation Current	Output (open circuit)
141A	5VDC–15VDC	10V	5 milliamperes	± 500 millivolts
141B	10VDC–28VDC	24V	10 milliamperes	± 1000 millivolts

Cable, Weight, Case

Electrical Connection	2 foot multiconductor cable
Weight	30 grams (not including cable)
Case	Stainless steel. O-ring sealed

Ordering Information

Specify:	Model 141A or Model 141B
Specify G Range:	Nominal Range (±specific g)
Specify:	Excitation voltage for calibration (If non-standard, at extra charge)

Specifications subject to change without notice

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