SOLUTION

MAE106 Mechanical Systems Laboratory

Quiz for Laboratory Exercise 4: Vibration I - Lightly Damped Second Order Systems

In lab 4, you measured the impulse and frequency responses of a beam.

10 1. Write in the box the differential equation that describes the beam dynamics, where:
   m is the mass
   c is the damping
   k is the stiffness
   f is the input force
   x is the beam deflection

$$m\ddot{x} + c\dot{x} + kx = f$$

10 2. What is the transfer function for the beam in terms of m, c, and k, given force as input and position as output?

$$\frac{x}{f} = H(s) = \frac{1}{ms^2 + cs + k} \qquad (ms^2 + cs + k)x = f$$

10 3. How would you calculate the (undamped) natural frequency of the beam, given m, c, and k?

$$\omega_n = \sqrt{\frac{k}{m}}$$

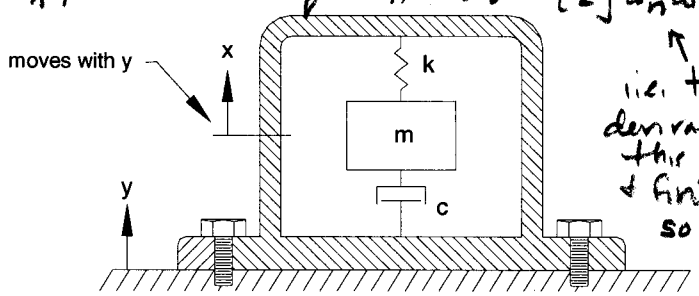
10 4. How did you measure the natural frequency of the beam in lab?

Either answer is OK — measured period between oscillations for impulse response $f_n = \frac{1}{\text{period}}$ / changed motor speed (i.e. forcing frequency) until beam resonated

10 5. Was the beam overdamped, critically damped, or underdamped? (circle one)

10 6. Given the equation of the transfer function for the beam, how would you mathematically solve for the exact resonant frequency ω_r for the beam? (You don't need to do it, just briefly explain how). at ω_r, scaling factor = $|G(j\omega)|$ is maximal ⇒ denominator of $|G(j\omega)|$ is minimal ⇒ find minimum of denominator of transfer fcn, evaluated at $s = j\omega$ ⇒ find minimum of $|(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2|^2$ ⇒ find min of $(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2$ (i.e. take derivative of this eqn & find ω so derivative = 0)

10 7. A schematic representation of the accelerometer used in the experiments is shown in the figure.



You found the transfer function of the accelerometer to be: $G(s) = \frac{-m}{ms^2 + cs + k}$

What are the input and output for the transfer function?

Input: acceleration

Output: displacement of the spring ($\frac{1}{2}$ credit for "voltage")

10 8. How is the output measured inside the accelerometer?

A capacitive sensor measures displacement of the spring

10 9. The accelerometer does not provide an accurate output signal for high input frequencies because it acts like a low pass filter.

10 10. In lab you calibrated the accelerometer by finding the calibration coefficient. The calibration coefficient relates acceleration input to voltage output.