MAE 106 Lab 3 Quiz and Midterm Exam
Winter 2005

Lab 3 Quiz = 100 pts

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Department of Mechanical and Aerospace Engineering

Part 1: Lab 3 Quiz
In Lab 3, you built a motor velocity controller, using a proportional feedback control.

15 1. Write the velocity control law that you used for the motor in the box, where:

u = the control input into the motor
K = feedback gain
ω_{actual} = actual motor speed
ω_d = desired motor speed

$\boxed{u = -K(\omega_{actual} - \omega_d)}$

$u = K(\omega_d - \omega_{actual})$ $\omega = K(\omega_{actual} - \omega_d)$ OK ALSO

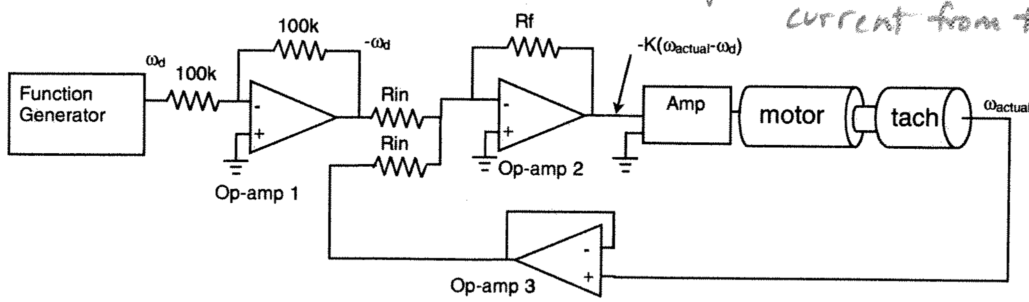
10 2. For the Lab 3 motor amplifier, the motor torque was proportional to the input.

24 3. Below is the control circuit that you used to implement P-type velocity control. Briefly explain the purpose of each op-amp.

Op Amp 1: Multiplies ω_d by -1 (inverts ω_d)

Op Amp 2: Implements control law $u = -K(\omega_{actual} - \omega_d)$

Op Amp 3: Buffers tachometer so that rest of circuit does not draw current from tach.



24 4. Fill out the below chart based on your experience in lab:

Increasing R_f will (circle one)		
Increase or decrease	f_{3db}	Cutoff frequency
Increase or decrease	$\tau = 1/\omega_{3db}$	Time constant
Increase or decrease	e_{ss}	Steady state error

15 5. The controlled motor behaved like what kind of filter? Low pass filter

12 6. At the cutoff frequency of the motor, the output amplitude was .707 of the input amplitude and the output waveform lagged the input by 45 degrees.

Part 2: Midterm

Problem 1 (10 Pts Extra Credit)

An oscilloscope is used to measure this:

2 Answer b a) resistance b) voltage c) current d) power

The time constant of a first-order system tells when the output has gotten how far along the way to its final value?

2 Answer c a) 37% b) 10% c) 63% d) 90%

If you put a sine wave into a linear system, you get the following out

2 Answer d
a) square wave
b) sine wave at different frequency
c) triangle wave
d) sine wave at same frequency, scaled and shifted

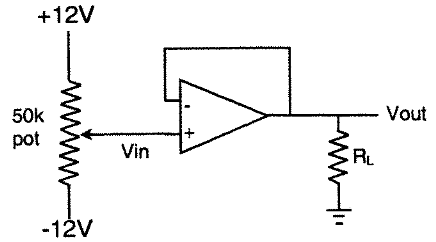
A filter scales a sinusoidal input. The amount of scaling is determined by:

2 Answer a
a) the magnitude of the transfer function, evaluated at $s=j\omega$
b) the magnitude of the transfer function, evaluated at $s = \omega$
c) the phase of the transfer function, evaluated at $s=j\omega$

A low pass filter attenuates

2 Answer b
a) low frequencies
b) high frequencies
c) a band of frequencies

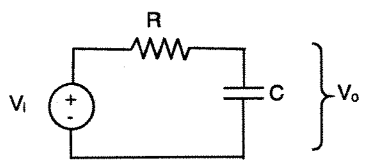
Problem 2 (25 pts)
How close is V_{out} to V_{in} for the following voltage follower circuit, if the op-amp gain is 1,000? (Hint, use the fact that $V_o = K(V_+ - V_-)$ for the op amp)



$$V_{out} = K(V_+ - V_-)$$
$$= K(V_{in} - V_{out})$$
$$V_{out}(1+K) = K V_{in}$$
$$V_{out} = \frac{K}{1+K} V_{in} = \frac{1000}{1001} V_{in}$$

Problem 3 (25 pts)
How does the following circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{2\pi} = 0.16$ Hz.

Assume R = 1 kiloohm and C=1milliFarad.



Transfer function using impedances:

$$V_o = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_i = \frac{1}{1+RCs} V_i$$

$$H(s) = \frac{1}{1+RCs}$$

5 pts $\rightarrow$ $$H(j\omega) = \frac{1}{1+RCj\omega}$$

$\omega = 2\pi f$

$= (2\pi)(\frac{1}{2\pi}) = 1$ rad/sec

$RC = 1$ sec

$$\text{Scaling} = |H(j\omega)| = \frac{1}{\sqrt{1+(RC\omega)^2}}$$

$$= \boxed{\frac{1}{\sqrt{2}} = .707} \leftarrow 10 \text{ pts}$$

$$\text{phase} = 0 - \tan^{-1}\frac{RC\omega}{1} = -\tan^{-1} RC\omega = -\tan^{-1} 1$$

$$\boxed{= 45° \text{ or } \frac{\pi}{4}}$$

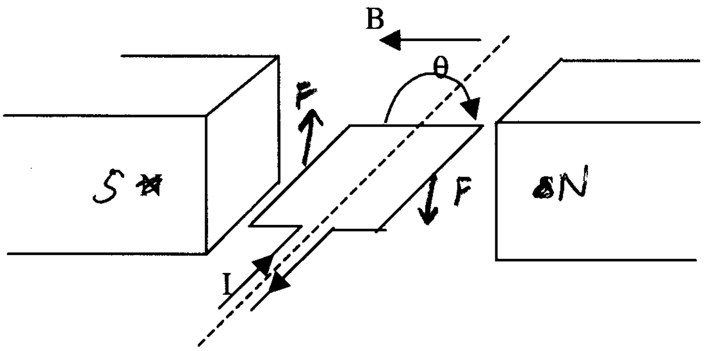
10 pts $\rightarrow$

Problem 4: 25 pts

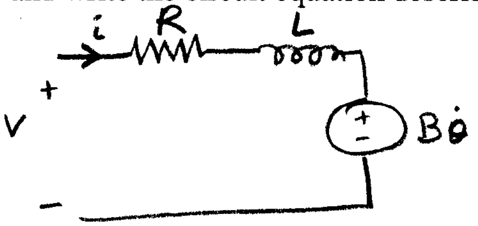
5 a. Shown below is a diagram of a DC brushed motor. Assume that the commutation stops working, such that current flows only in the direction shown. At what angle θ will the armature come to rest? Assume the armature is initially at $\theta = 0°$ as shown when the commutation fails, and that positive θ is defined clockwise looking into the page, as shown.

$F = i\vec{\ell} \times \vec{B}$

$\theta = 90°$



5 b. For the rest of this problem, assume the commutation is working. Draw the circuit model, and write the circuit equation describing the motor:



$V = Ri + L\frac{di}{dt} + B\dot{\theta}$

15 b. Solve this differential equation for the current through the motor as a function of time when:

- the shaft of the motor is held fixed
- a constant voltage v is applied across the motor at time = 0
- the initial current i(t = 0) through the inductor is zero

Shaft fixed $\Rightarrow \dot{\theta} = 0$

$L\frac{di}{dt} + Ri = V$

Soln to homog. eqn:

$\frac{di}{dt} = -\frac{R}{L} i \Rightarrow i = Ae^{-t/\tau} \quad \tau = \frac{L}{R}$

Part. Soln:

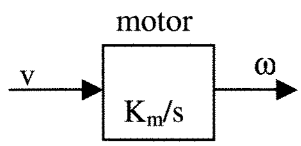
$i = \frac{V}{R}$

Total Soln: $i = Ae^{-t/\tau} + \frac{V}{R}$ but $i(0) = 0$

$\Rightarrow \boxed{i = \frac{V}{R}\left(1 - e^{-t/\tau}\right) \quad \tau = \frac{L}{R}}$

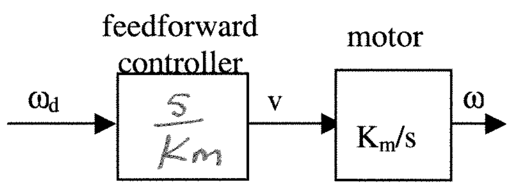
Problem 5: 25 pts

1) You want to control the speed of a motor. You are using a current amplifier with the motor, so the speed is related to the input voltage to the current amplifier by the following transfer function:

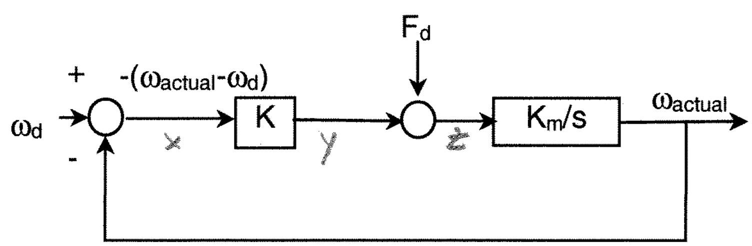


where v is the voltage input to the motor and ω is the angular velocity of the shaft and K_m is a constant.

10 pts a) Shown below is a block diagram of an open-loop (i.e. feedforward) controller for the motor, where ω_d is the desired output of the motor. What transfer function should the controller box have to make the output equal the desired output? Write this function controller box.



15 pts b) One of the major benefits of feedback is its ability to cancel the effects of unmodeled "disturbances". Assume you build a feedback controller, but there is a disturbance force F_d affecting the motor:



Derive an expression that relates ω_{actual} to ω_d and F_d, then prove that the disturbance is cancelled if K is large enough.

$x = w_d - w_{actual}$

$y = Kx = K(w_d - w_{actual})$

$z = y + F_d$

$w_{actual} = \frac{K_m}{s} z = \frac{K_m}{s}\left(K(w_d - w_{actual}) + F_d\right)$

$s\, w_{actual} = K_m K w_d - K_m K w_{actual} + K_m F_d$

$\Rightarrow w_{actual}(s + K_m K) = K_m K w_d + K_m F_d$

10 pts $\boxed{w_{actual} = \frac{K_m K}{s + K_m K} w_d + \frac{K_m}{s + K_m K} F_d}$

$\boxed{\text{as } K \to \infty \quad w_{actual} \to w_d}$ 5 pts