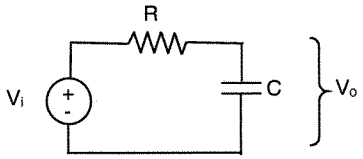
SOLUTION VERSION "A"

MAE 106 Post-Laboratory Quiz
Laboratory Exercise #2: Electrical Filters and First-Order Systems

In Lab 1, you learned how simple RC low-pass and high-pass filter work, and how first-order systems behave in the time and frequency domains.

50 pts 1. Sketch V_o if V_i is a 1 Volt square wave input at 0.5 Hz for the low-pass filter circuit shown below. Assume R = 1 kiloohm and C=1milliFarad. Assume that square wave turns on at t = 0.0 sec.

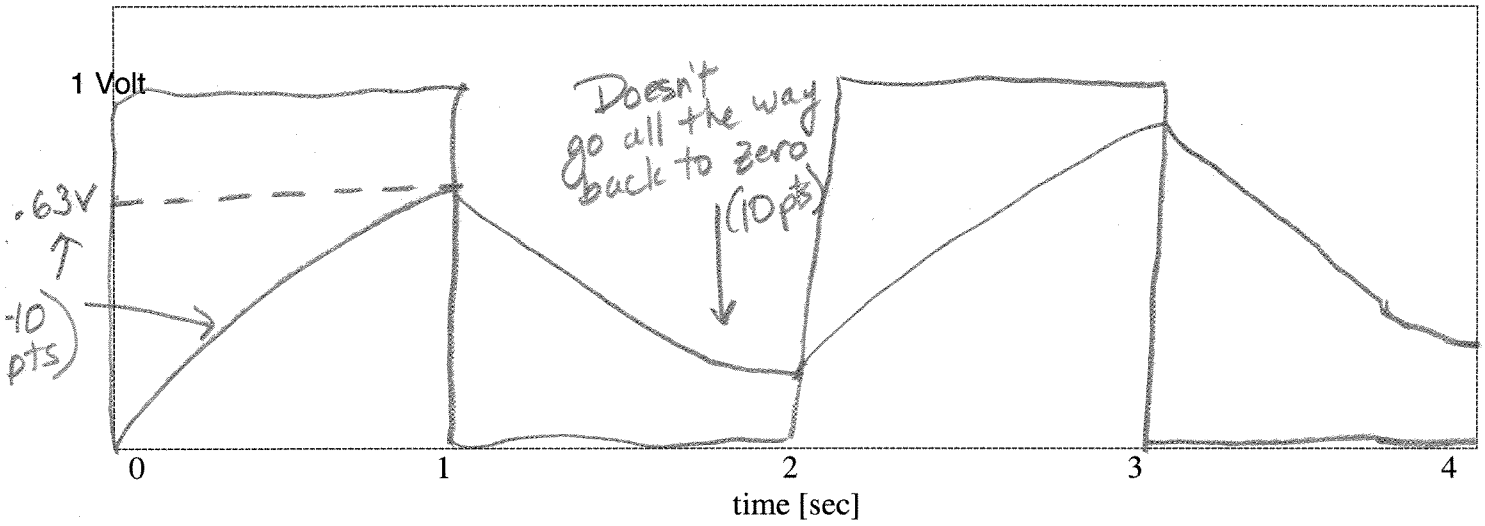


$\tau = RC = (1\times10^{3})(1\times10^{-3}) = 1$ sec

$V_o = V_i(1 - e^{-t/\tau})$ ← SOLN

$-V_i + Ri + V_o = 0$

$-V_i + RC\frac{dV_o}{dt} + V_o = 0$



50 pts 2. How does the circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{2\pi} = 0.16$ Hz?

$\text{Scaling} = |H(j\omega)| = \frac{1}{\sqrt{1+(\omega\tau)^2}} = \frac{1}{\sqrt{1+1}}$

$\omega = 2\pi f = (2\pi)\frac{1}{2\pi} = 1$ rad/sec

$\text{phase} = \phi_H(j\omega) = -\tan^{-1}\frac{1}{1}$

(25) Scaling: $= \frac{1}{\sqrt{2}} = .707$

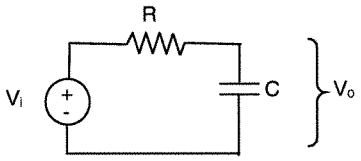
(25) Phase Shift $= -\tan^{-1}1 = -45^\circ = .78$ rad $= \frac{\pi}{4}$ rad

SOLUTION VERSION "B"

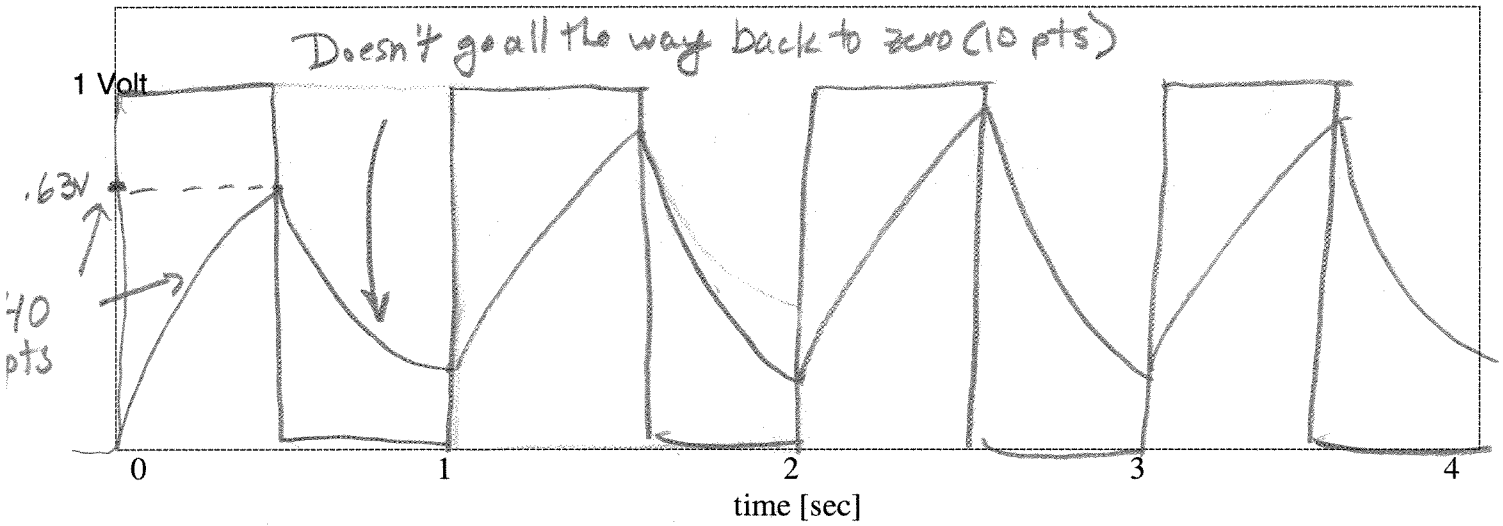
MAE 106 Post-Laboratory Quiz
Laboratory Exercise #2: Electrical Filters and First-Order Systems

In Lab 1, you learned how simple RC low-pass and high-pass filter work, and how first-order systems behave in the time and frequency domains.

50 3. Sketch V_o if V_i is a 1 Volt square wave input at 1.0 Hz for the low-pass filter circuit shown below. Assume R = 0.5 kilo-ohm and C=1milliFarad. Assume that square wave turns on at t = 0.0 sec.



$\tau = RC = .5\times10^{3}\times 1\times10^{-3} = .5$ sec



50 4. How does the circuit filter a low frequency input? Specifically, find what the resulting scaling and phase-shift would be for an input sinusoid with a frequency of $\frac{1}{4\pi} = 0.08$ Hz?

$H(s) = \frac{1}{1+\tau s}$

$\omega = 2\pi f = 2\pi \frac{1}{4\pi} = \frac{1}{2}$

scaling $= |H(j\omega)| = \left|\frac{1}{1+\tau j\omega}\right| = \frac{1}{\sqrt{1+(\omega\tau)^2}} = \frac{1}{\sqrt{1+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2}} = \frac{1}{\sqrt{1+\frac{1}{8}}} = \frac{1}{\sqrt{1.125}}$

phase shift $= \phi_H(j\omega) = 0^\circ - \tan^{-1}\omega\tau = 0 - \tan^{-1}\frac{1}{4}$

25 Scaling: $\frac{1}{\sqrt{1.125}} = .94$

25 Phase Shift: $-\tan^{-1}.25 = -14^\circ$ or $-.245$ rad