LAB #2 report. MAE 106. UCI. Winter 2005

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1 Answer 1.

Explain time constant: This is the time the output will take to reach 63% of its final value.

2 Answer 2.

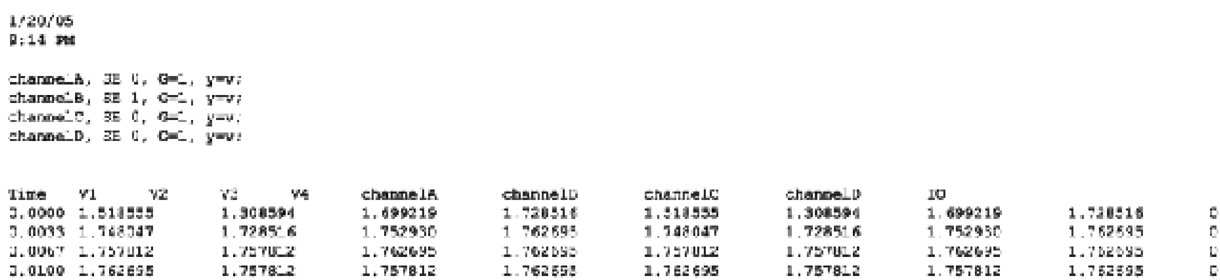
Cut-off frequency is the frequency at which the output amplitude (Such as voltage) is 70.1% of the input. This corresponds to a drop of $-3db$ from the input. It is also the frequency at which the output power is 50% that of the input. This frequency is usually taken as the boundary frequency between the highpass region and the low pass region of the frequency response plot.

3 Answer 3

A heavy object has large inertia. Which means it will take time for it to accelerate when subjected to the same force as compared to an object of low mass. When the force fluctuates very quickly, the object will be slow to react due to its high mass, and by the time it starts to move in response to the force, the force will change its direction quickly, and the object will have to start to reverse its direction again in the direction of the force, and will again be slow in doing this new movement. So this mean the object will have a small motion amplitude of the same frequency as the input force. This is a low pass filter.

4 Answer 4.

I collected the data using LabJack. This below shows the first few lines of the text file collected:



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9:14 PM

channelA, SE 0, G=1, y=v;
channelB, SE 1, G=1, y=v;
channelC, SE 0, G=1, y=v;
channelD, SE 0, G=1, y=v;

Time    V1       V2      V3      V4     channelA    channelB    channelC    channelD    IO
0.0000  1.518555         1.308594       1.699219    1.728516    1.518555    1.308594    1.699219    1.728516    0
0.0033  1.748047         1.728516       1.752930    1.762695    1.748047    1.728516    1.752930    1.762695    0
0.0067  1.757812         1.757812       1.762695    1.762695    1.757812    1.757812    1.762695    1.762695    0
0.0100  1.762695         1.757812       1.757812    1.762695    1.762695    1.757812    1.757812    1.762695    0
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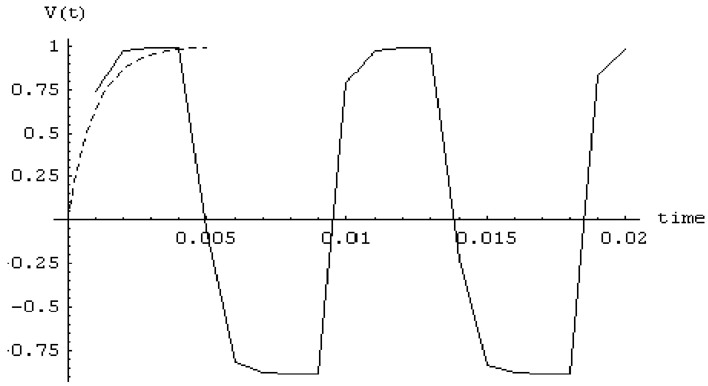
Now I wrote a small script to plot the data and the theoretical response $V_{in}\left(1-e^{-\frac{t}{RC}}\right)$ on the same plot. this is the result. I normalized both amplitudes to 1.



5 Answer 5.

For the circuit in figure 1, the ODE is $V_o = V_i - CR\frac{dV_o}{dt}$. Take Laplace transform, we get $V_o(s) = V_i(s) - CRsV_o(s) \Rightarrow V_o(s)[1+CRs] = V_i(s) \Rightarrow \boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{1+CRs}}$ hence $\boxed{H(j\omega) = \frac{1}{1+jCR\omega}}$ hence, $|H| = \frac{1}{\sqrt{1+(CR\omega)^2}}$ and phase is $\phi(H) = -\tan^{-1}(\omega CR) = \ -\tan^{-1}(\omega CR)$

Hence for $C = 10^{-6}F$, and $R = 10^3$ we get $|H| = \frac{1}{\sqrt{1+\omega^2(10^{-6}10^3)^2}} = \boxed{\frac{1}{\sqrt{1+\omega^2 1.0\times10^{-6}}}}$

and $\phi(H) = \ \tan^{-1}\left(\omega 10^{-6}\times 10^3\right) = \boxed{-\tan^{-1}(0.001\,\omega)}$

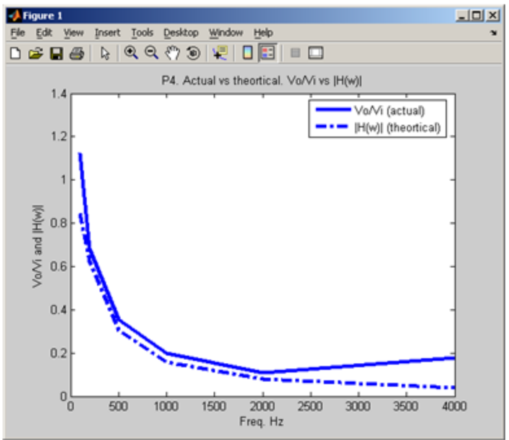
The following is the data collected for **P4,** and the theoretical data based on above Laplace transform. This is a **low pass filter.**

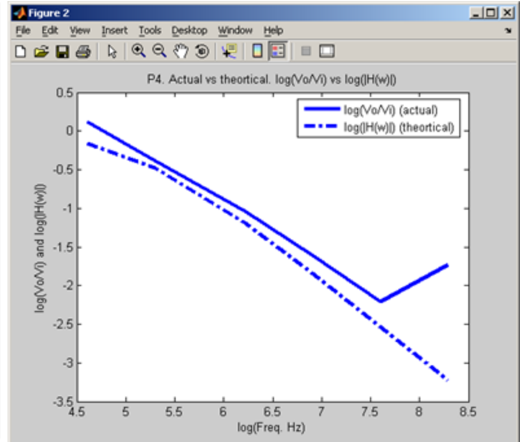
Input Freq. (Hz)	Vpp input	Vpp output	Δt	T	$\Phi = \frac{\Delta t}{T}2\pi\ \frac{180}{\pi}$(degrees)
100	590 mV	662 mV	1.04 ms	10 ms	$\frac{-1.04}{10}2\pi\ \frac{180}{\pi}$ = -37.44^0
200	1.7 V	1.156 V	740 μs	4.9 ms	$\frac{-740\times10^{-6}}{4.9\times10^{-3}}2\pi\ \frac{180}{\pi}$ = -54.367
500	1.59 V	562 mV	390 μs	1.99 ms	$\frac{-390\times10^{-6}}{1.99\times10^{-3}}2\pi\ \frac{180}{\pi} = -70.553$
1 K	1.562 V	312 mV	244 μs	990 μs	$\frac{-244}{990}2\pi\ \frac{180}{\pi} = -88.727$
2 K	1.56 V	171 mV	126 μs	515 μs	$\frac{-126}{515}2\pi\ \frac{180}{\pi} = -88.078$
4 K	2.8 V	500 mV	65 μs	242 μs	$\frac{-65}{242}2\pi\frac{180}{\pi} = -96.694$

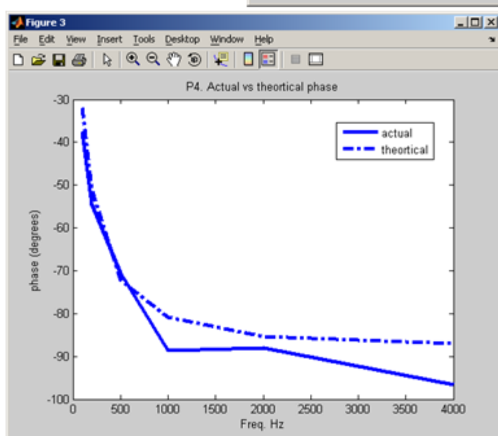
Theortical:

Input Freq. (Hz)	$\lvert H\rvert$	$\phi(H)$ (degrees)
100	$\frac{1}{\sqrt{1+(2\pi\times100)^2 1.0\times10^{-6}}} = 0.84673$	-tan$^{-1}(0.001\times2\pi\times100)\ \frac{180}{\pi} = -32.14^0$
200	$\frac{1}{\sqrt{1+(2\pi\times200)^2 1.0\times10^{-6}}} = 0.62268$	-tan$^{-1}(0.001\times2\pi\times200)\ \frac{180}{\pi} = -51.488$
500	$\frac{1}{\sqrt{1+(2\pi\times500)^2 1.0\times10^{-6}}} = 0.30331$	-tan$^{-1}(0.001\times2\pi\times500)\ \frac{180}{\pi}$ = -72.343
1 K	$\frac{1}{\sqrt{1+(2\pi\times1000)^2 1.0\times10^{-6}}} = 0.15718$	-tan$^{-1}(0.001\times2\pi\times1000)\ \frac{180}{\pi} = -80.957$
2 K	$\frac{1}{\sqrt{1+(2\pi\times2000)^2 1.0\times10^{-6}}} = 7.9327\times10^{-2}$	$-\tan^{-1}(0.001\times2\pi\times2000)\ \frac{180}{\pi} = -85.45$
4 K	$\frac{1}{\sqrt{1+(2\pi\times4000)^2 1.0\times10^{-6}}} = 3.9757\times10^{-2}$	$-\tan^{-1}(0.001\times2\pi\times4000)\ \frac{180}{\pi} = -87.721$

I now wrote a script to plot the needed plots as required by P4. This is the result. This shows that the amplitude plot involving the log is more clear and it shows the low pass filter, this is because when using the log scaling, the becomes straight lines.







The following is the data collected for **P5.**

Since $q = CV_c$ for the capacitor, and for the circuit in figure 3, we get $V_{in} - V_o - \frac{q}{C} = 0$. Take derivative, we get $\frac{dV_i}{dt} - \frac{dV_o}{dt} - \frac{1}{c}\frac{dq}{dt} = 0$, but $\frac{dq}{dt} = i = \frac{V_o}{R}$, hence ODE becomes $\frac{dV_i}{dt} - \frac{dV_o}{dt} - \frac{1}{C}\frac{V_o}{R} = 0$, take Laplace transform we get $sV_i(s) - sV_o(s) - \frac{V_o(s)}{RC} \Rightarrow sV_i(s) = V_o(s)\left[s + \frac{1}{RC}\right] \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{s}{s+\frac{1}{RC}} = \boxed{\frac{sRC}{sRC+1}}$

Let $s = j\omega$, $\Rightarrow H(j\omega) = \frac{j\omega RC}{j\omega RC+1} \Rightarrow |H| = \frac{\omega RC}{\sqrt{(\omega RC)^2+1}}$ and $\phi(H) = \frac{\pi}{2} - \left(\tan^{-1}\omega RC\right)$

Hence for $C = 10^{-6}F$, and $R = 10^3$ we get $|H| = \frac{0.001\omega}{\sqrt{(0.001\omega)^2+1}} = \boxed{\frac{0.001\omega}{\sqrt{1+\omega^2 1.0\times10^{-6}}}}$

and $\phi(H) = \boxed{\frac{\pi}{2} - \left(\tan^{-1}0.001\omega\right)}$

This is a **high pass filter**

Input Freq. (Hz)	Vpp output	Vpp input	Δt	T	$\Phi = \frac{\Delta t}{T}2\pi\frac{180}{\pi}$ (degrees)
10	800 mV	10 V	38 ms	97 ms	$\frac{38}{95}2\pi\frac{180}{\pi} = 144.0$
50	3 V	9.8 V	15 ms	20 ms	$\frac{15}{20}2\pi\frac{180}{\pi} = 270.0$
100	5 V	9.18 V	8.24 ms	10 ms	$\frac{8.24}{10}2\pi\frac{180}{\pi} = 296.64$
200	6.6 V	8.25 V	4.34 ms	5.07 ms	$\frac{4.34}{5.07}2\pi\frac{180}{\pi} = 308.17$
400	7.3 V	7.7 V	2.4 ms	2.52 ms	$\frac{2.4}{2.52}2\pi\frac{180}{\pi} = 342.86$
1000	7.47 V	7.47 V	48 μs	989 μs	$\frac{48}{989}2\pi\frac{180}{\pi} = 17.472$
2000	7.57 V	7.47 V	8 μs	500 μs	$\frac{8}{500}2\pi\frac{180}{\pi} = 5.76$
4000	7.5 V	7.3 V	4 μs	247 μs	$\frac{4}{247}2\pi\frac{180}{\pi} = 5.8300$

Theortical:

Input Freq. (Hz)	$\lvert H\rvert$	$\phi(H) = \frac{\pi}{2} - \left(\tan^{-1} 0.001\omega\right)$
10	$\frac{0.001\times 2\pi\times 10}{\sqrt{1+(2\pi\times 10)^2 1.0\times 10^{-6}}} = 6.2708\times 10^{-2}$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 10)\right)\frac{180}{\pi} = 86.405^0$
50	$\frac{0.0012\pi\times 50}{\sqrt{1+(2\pi\times 50)^2 1.0\times 10^{-6}}} = 0.17983$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 50)\right)\frac{180}{\pi} = 72.559$
100	$\frac{0.0012\pi\times 100}{\sqrt{1+(2\pi\times 100)^2 1.0\times 10^{-6}}} = 0.31921$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 100)\right)\frac{180}{\pi} = 57.858$
200	$\frac{0.0012\pi\times 200}{\sqrt{1+(2\pi\times 200)^2 1.0\times 10^{-6}}} = 0.46949$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 200)\right)\frac{180}{\pi} = 38.512$
400	$\frac{0.0012\pi\times 400}{\sqrt{1+(2\pi\times 400)^2 1.0\times 10^{-6}}} = 0.55749$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 400)\right)\frac{180}{\pi} = 21.697$
1000	$\frac{0.0012\pi\times 1000}{\sqrt{1+(2\pi\times 1000)^2 1.0\times 10^{-6}}} = 0.59254$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 1000)\right)\frac{180}{\pi} = 9.0431$
2000	$\frac{0.0012\pi\times 2000}{\sqrt{1+(2\pi\times 2000)^2 1.0\times 10^{-6}}} = 0.59811$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 2000)\right)\frac{180}{\pi} = 4.5499$
4000	$\frac{0.0012\pi\times 4000}{\sqrt{1+(2\pi\times 4000)^2 1.0\times 10^{-6}}} = 0.59953$	$\left(\frac{\pi}{2} - \tan^{-1}(0.001\times 2\pi\times 4000)\right)\frac{180}{\pi} = 2.2785$