MAE91 Summer 2004 - Quiz 3
Dr. H. Susan Thou
Closed book and notes - 20 minutes
Name: $\square$ NASSER $A B B A S I$ ID: $\qquad$
For each of the following processes draw a P -v and T -v diagrams. If there is a stop in the problem, assume that the process hits the stop and continues. Diagrams all show initial or ambient state. (1 point per diagram)


Heat is added to the system.
Springs constants are the same.
$x_{1}$ defined $x_{2}$ undefined


Heat leaves the system.
$x_{1}$ undefined, $x_{2}=1$


Mass is added to the system.
Process is adiabatic. $W \circ Q$.
$x_{1}$ defined, $x_{2}=0$
5)


Force added which pushes piston down.
Cylinder is refrigerated to a constant T.
$x$ is always undefined and greater than 1 .
6)


Heat is added to the system.
Initially a compressed liquid.


Heat is added to the system.
$x$ is always defined. $\underline{P}_{l}<P_{\text {float }}$
$P_{\text {float }}$ is the pressure needed to float piston


## Bonus Problem (2 points per diagram)



## Multi-Stage Process

- Piston is pushed down to bottom stop while maintaining constant T.
- When piston reaches bottom stop force ceases and LARGE amount of heat is added to system.
- Pressure is now greater than initial pressure and pressure after compression.
- Piston then cools back to ambient conditions.

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Heat is added to the system. Springs constants are the same. $x_{1}$ defined, $x_{2}$ undefined



Mass is added to the system. Process is adiabatic. $x_{1}$ defined, $x_{2}=0$
5)


Force added which pushes piston down.
Cylinder is refrigerated to a constant T.
$x$ is always undefined and greater than 1 .


Heat is added to the system. Initially a compressed liquid. Finally a superheated vapor.


Heat is added to the system. $x$ is always defined. $P_{1}<P_{\text {float }}$



## Multi-Stage Process

- Piston is pushed down to bottom stop while maintaining constant T .
- When piston reaches bottom stop force ceases and LARGE amount of heat is added to system.
- Pressure is now greater than initial pressure and pressure after compression.
- Piston then cools back to ambient conditions.


## Discussion Questions - Set \#2

## 1. Problem Statement:

An insulated cylmder fitted with a piston contains $R-12$ at $25^{\circ} \mathrm{C}$ with a quality of $90 \%$ and a volume of 45 L . The piston is allowed to move, and the $\mathrm{R}-12$ expands until it exists as saturated vapor. During this process the R-12 does 7.0 kJ of work against the piston. Determine the final temperature, assuming the process is adiabatic.

2. Assumptions and Givens:

Take C.V. to be the R-12
Process is adiabatic
State 1: $\quad T_{1}, x_{1}, V_{1}$
Process: $\quad{ }_{1} W_{2}$ Work is done
State 2: $\quad x_{2}$
Find: $T_{2}$

## 3. Fundamental Laws:

Continuity:
$m_{1}=m_{2}=m$
Energy Equation (First Law):
$U_{2}-U_{1}+\frac{m\left(\vec{V}_{2}^{2}-\vec{V}_{1}^{2}\right)}{2}+m g\left(Z_{2}-Z_{1}\right)={ }_{1} Q_{2}-W_{1}$

$$
\begin{aligned}
v & =\frac{V}{m} \\
v & =v_{f}+x v_{f g} \\
u & =u_{f}+x u_{f g}
\end{aligned}
$$

## 4. Steps:

This is an adiabatic process, with no change in kinetic or potential energy. So:

$$
\begin{aligned}
& U_{2}-U_{1}={ }_{1} W_{2} \\
& m\left(u_{2}-u_{1}\right)={ }_{1} W_{2}
\end{aligned}
$$

Using Table B.1.1 and State 1:

$$
\begin{aligned}
& T_{1}, x_{1} \Rightarrow v_{f 1}, v_{g 1}, v_{f 8}, u_{f 1}, u_{g 1}, u_{f g 1} \\
& v_{1}=v_{f 1}+x_{1} v_{f g 1} \\
& m=\frac{V_{1}}{v_{1}} \\
& u_{1}=u_{f 1}+x_{1} u_{f g 1}
\end{aligned}
$$

For State 2 we only know 1 independent property: so we apply the energy equation:

$$
\begin{aligned}
& m\left(u_{2}-u_{1}\right)=W_{2} \Rightarrow u_{2} \\
& u_{2}=u_{g} \text { because } x_{2}=1
\end{aligned}
$$

Now we have two independent properties so we can use table B.2.1 to find $T_{2}$

## 5. Numerical Substitution:

$$
\begin{aligned}
& \text { Stace } 1:(\mathrm{T}, \mathrm{x}) \text { Tabel B. } 3.1= \\
& \mathrm{v}_{1}=0.000763+0.9 \times 0.02609=0.024244 \mathrm{~m}^{3} / \mathrm{kg} \\
& \mathrm{~m}=\mathrm{V}_{1} \mathrm{~N}_{\mathrm{L}}=0.0450 .024244=1.856 \mathrm{~kg} \\
& \mathrm{u}_{1}=59.21+0.9 \times 121.03=168.137 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

State 2: ( $\mathrm{x}=1,7$ ) We need one property information.

$$
\begin{aligned}
Q_{2}=0 & =n\left(u_{2}-u_{1}\right)+W_{2}=1.856 \times\left(u_{2}-168.137\right)+7.0 \\
& =\quad u_{2}=164.365 \mathrm{k} / \mathrm{kg}=u_{2} \text { at } T_{2}
\end{aligned}
$$

Table B.3.1 gives $\mathrm{u}_{\mathrm{g}}$ at different temperatures: $\mathrm{T}_{2} \cong-15^{\circ} \mathrm{C}$

### 5.57

## 1. Problem Statement:

A cylinder having a piston restramed by a linear spring (of spring constant $15 \mathrm{kN} / \mathrm{m}$ ) contains 0.5 kg of saturated vapor wates at $120^{\circ} \mathrm{C}$, as shown in Fig. P5.57. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is $0.05 \mathrm{~m}^{2}$, and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperanure in the cylinder and the heat transfer for the process.


2. Assumptions and Givens:
C.V. is water in the cylinder

State 1: $\quad T_{1}, x_{1}, k_{s}, A_{p}$
Process: Heat is transferred and the piston moves
State 2: $\quad P_{2}$
Find: $T_{2},{ }_{1} Q_{2}$

## 3. Fundamental Laws:

## Continuity:

$m_{1}=m_{2}=m$
Energy Equation (First Law):
$U_{2}-U_{1}+\frac{m\left(\vec{V}_{2}^{2}-\vec{V}_{1}^{2}\right)}{2}+m g\left(Z_{2}-Z_{1}\right)={ }_{1} Q_{2}-W_{1}$
4. Steps:

We can simplify the energy equation knowing that there is no KE or PE .
$U_{2}-U_{1}={ }_{1} Q_{2}-{ }_{1} W_{2}$
$m\left(u_{2}-u_{1}\right)={ }_{1} Q_{2}-{ }_{1} W_{2}$

Using Table B.1.1 and knowing two independent properties we can find:

$$
T_{1}, x_{1} \Rightarrow v_{1}=v_{g 1}, u_{1}=u_{g 1}
$$

From previous problems we know that for a spring attached to a piston:

$$
P_{2}=P_{1}+\frac{k_{s} m}{A_{p}^{2}}\left(v_{2}-v_{1}\right) \Rightarrow v_{2}
$$

Now we have two independent properties of state 2:

$$
P_{2}, v_{2} \Rightarrow v_{1}=T_{2}, u_{2}
$$

Now that we have the final temperature to find the heat transfer we use the energy equation. In this case since the relationship between $P$ and $V$ is linear we can use the following to calculate the work:

$$
W_{2}=\int P d V=\frac{1}{2}\left(P_{1}+P_{2}\right) m\left(v_{2}-v_{1}\right)
$$

Therefore, we now have all the information to find the heat transfer:

$$
{ }_{1} Q_{2}=m\left(u_{2}-u_{1}\right)+{ }_{1} W_{2}
$$

## 5. Numerical Substitution:

State I: $(T, x)$ Table B.1.1 $\Rightarrow v_{1}=0.89186 \mathrm{~m}^{3} / \mathrm{kg}, \quad u_{1}=2529.2 \mathrm{~kJ} / \mathrm{kg}$
Process: $\quad P_{2}=P_{1}+\frac{\mathrm{k}_{\mathrm{s}} \mathrm{m}}{\mathrm{A}_{\mathrm{p}}^{2}}\left(\mathrm{v}_{2}-\mathrm{v}_{1}\right)=198.5+\frac{15 \times 0.5}{(0.05)^{2}}\left(\mathrm{v}_{2}-0.89186\right)$
State 2: $\quad \mathrm{P}_{2}=300 \mathrm{kPa}$ and on the process curve (see above equation).

$$
\begin{aligned}
& \Rightarrow \quad v_{2}=0.89186+(500-198.5) \times\left(0.05^{2} / 7.5\right)=0.9924 \mathrm{~m}^{3} / \mathrm{kg} \\
& \text { (P. v) Table B.1.3 }=T_{2}=803^{\circ} \mathrm{C} ; \quad u_{2}=3668 \mathrm{~kJ} / \mathrm{kg} \\
& W_{12}=\int P d V=\left(\frac{P_{1}+P_{2}}{2}\right) \mathrm{m}\left(\mathrm{v}_{2}-\mathrm{v}_{1}\right) \\
& \quad=\left(\frac{198.5 \div 500}{2}\right) \times 0.5 \times(0.9924-0.89186)=17.56 \mathrm{~kJ} \\
& { }_{1} Q_{2}=m\left(u_{2}-u_{1}\right)+{ }_{1} W_{2}=0.5 \times(3668-2529.2)+17.56=587 \mathrm{~kJ}
\end{aligned}
$$

## 1. Problem Statement:

The cylinder volume below the constant loaded piston has two compartments $A$ and $B$ filled with water. A has 0.5 kg at $200 \mathrm{kPa}, 150^{\circ} \mathrm{C}$ and B has 400 kPa with a quality of $50 \%$ and a volume of $0.1 \mathrm{~m}^{3}$. The valve is opened and heat is transferred so the water comes to a uniform state with a total volume of $1.006 \mathrm{~m}^{3}$.
a) Find the sotal mass of water and the total initial volume.
b) Find the work in the process
c) Find the process heat transfer.



## 2. Assumptions and Givens:

CV is water in A and B
State 1A: $\quad m_{1 A}, P_{1 A}, T_{1 A}$
State 1B: $\quad P_{1 B}, x_{1 B}, V_{1 B}$
Process: Heat is transferred and valve is opened
State 2: $\quad V_{2}$
Find: See Problem statement
3. Fundamental Laws:

Continuity:
$m_{1 A}+m_{1 B}=m_{2}=m$

Energy Equation (First Law):
$U_{2}-U_{1}+\frac{m\left(\vec{V}_{2}^{2}-\vec{V}_{1}^{2}\right)}{2}+m g\left(Z_{2}-Z_{1}\right)={ }_{1} Q_{2}+W_{2}$
$v=\frac{V}{m}$
$v=v_{f}+x v_{f g}$
$u=u_{f}+x u_{f g}$

## 4. Steps:

Simplifying the Energy equation gives us:
$U_{2}-U_{1}={ }_{1} Q_{2}-{ }_{1} W_{2}$
$m_{2} u_{2}-\left(m_{1 A} u_{1 A}+m_{1 B} u_{1 B}\right)={ }_{1} Q_{2}-W_{1}$
$m_{2} u_{2}-m_{1 A} u_{1 A}-m_{1 B} u_{1 B}=Q_{1} Q_{2}-W_{2}$
a) If $V_{2}>V_{B}$ then the piston will never hit the stops and P is constant.

Using Table B.1.3 we can find the properties of State 1A:
$T_{1 A}, P_{1 A} \Rightarrow v_{1 A}, u_{1 A}$
$V_{1 A}=m_{1 A} v_{1 A}$
$\therefore V_{1}=V_{1 A}+V_{1 B}$
Now we can compare $V_{1}$ to $V_{2}$
Using Table B.1.2 we can find the properties of State 1B:
$P_{1 B}, x_{1 B} \Rightarrow v_{1 A}, u_{1 A}$ using quality relationships
$m_{1 B}=V_{1 B} / v_{1 B}$
$\therefore m_{t o t}=m_{1 A}+m_{1 B}=m_{2}$
$\therefore v_{2}=V_{2} / m_{2}$
Now we can find out all the information about State 2, using table B.1.3 $P_{2}, v_{2} \Rightarrow T_{2}, u_{2}$
b) Since the final volume is bigger than the initial volume, that means the piston floats so it is constant pressure. There the work is:

$$
W_{2}=\int P d V=P \Delta V=P_{\text {foout }} \Delta V=P_{2}\left(V_{2}-V_{1}\right)
$$

c) Finally, using the energy equation we solve for heat.

$$
{ }_{1} Q_{2}=W_{1} W_{2}+m_{2} u_{2}-m_{1 A} u_{1 A}-m_{1 B} u_{1 B}
$$

## 5. Numerical Substitution:

State A1: Sup. vap. Table B. $1.3 \mathrm{v}=0.95964 \mathrm{~m}^{3} / \mathrm{kg}, \mathrm{u}=2576.9 \mathrm{kJikg}$

$$
=\Rightarrow \mathrm{V}=\mathrm{mv}=0.5 \times 0.95964=0.47982
$$

State B1: Table B. $1.2 \quad \mathrm{v}=(\mathrm{I}-\mathrm{x}) \times 0.001084+\mathrm{x} \times 0.4625=0.2318 \mathrm{~m}^{3} / \mathrm{kg}$

$$
\Rightarrow \quad \mathrm{m}=\mathrm{V} / \mathrm{v}=0.4314 \mathrm{~kg}
$$

$$
u=604.29+0.5 \times 1949.3=1578.9 \mathrm{~kJ} \mathrm{~kg}
$$


Table B.1.3 $\Rightarrow$ close to $T_{2}=200^{\circ} \mathrm{C}$ and $u_{2}=2654.4 \mathrm{k} / \mathrm{kg}$
So now

$$
V_{1}=0.47982+0.1=0.5798 \mathrm{~m}^{3}, \mathrm{~m}_{1}=0.5+0.4314=0.9314 \mathrm{~kg}
$$

Since volume at state 2 is larger than initial volume piston goes up and the pressure then is coustant ( 200 kPa which floats piston)

$$
\begin{aligned}
{ }_{1} W_{2} & =P d V=P_{\text {lifi }}\left(V_{2}-V_{1}\right)=200(1.006-0.57982)=85.24 \mathrm{~kJ} \\
{ }_{1} Q_{2} & =m_{2} \mathrm{k}_{2}-\mathrm{m}_{1 A} \mathrm{u}_{1 \mathrm{~A}}-\mathrm{m}_{1 \mathrm{~B}} \mathrm{u}_{1 B}+{ }_{1} \mathrm{~W}_{2} \\
& =0.9314 \times 2654.4-0.5 \times 2576.9-0.4314 \times 1578.9+85.24=588 \mathrm{~kJ}
\end{aligned}
$$

