Discussion, MAE 91, 8/11/2004

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Contents

1 Problem 8.95

1.1 Statment

see book.

1.2 Assumptions

air is ideal gas
constant specific heat coefficients

1.3 Laws

${}_1Q_2 - W = m(u_2 - u_1)$
$du = C_{v0}\ dT$
$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ for adiabatic process
$PV = mRT$

1.4 Steps

Control mass energy equation gives

$${}_1Q_2 - W = m(u_2 - u_1)$$

Since adiabatic, then ${}_1Q_2 = 0$, hence

$$W = -m\,(u_2 - u_1)$$
$$W = -m\ du$$

But $du = C_{v0}\ dT$, hence above becomes

$$\begin{aligned} W &= -m\ C_{v0}\ dT \\ &= -m\ C_{v0}\ \ (T_2 - T_1) \end{aligned} \tag{1}$$

Need to find T_2
Since adiabatic, then

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

Where $k = \frac{C_{p0}}{C_{v0}}$
So equation (1) becomes

$$\begin{aligned} W &= -m\ C_{v0}\ \left(T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - T_1\right) \\ &= -m\ C_{v0}\ T_1 \left(\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right) \end{aligned} \tag{2}$$

Mass m can be found from ideal gas law. $PV = mRT$, hence $m = \frac{PV}{RT}$, so equation (2) becomes

$$\begin{aligned} W &= -\frac{P_1 V_1}{R T_1}\ C_{v0}\ T_1 \left(\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right) \\ W &= -\frac{P_1 V_1}{R}\ C_{v0}\ \left(\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1\right) \end{aligned} \tag{3}$$

1.4.1 Numerical

For air, from table A.5, use $C_{v0} = 0.717$ KJ/Kg-K, $K = 1.4$, $R = 0.287$, so equation (3) becomes

$$\begin{aligned} W &= -\frac{(15 \times 10^3)\,(10 \times 10^{-6})}{0.287}\ \ (0.717)\ \left(\left(\frac{200}{15000}\right)^{\frac{1.4-1}{1.4}} - 1\right) \\ &= 0.265\,60 \quad \text{KJ} \end{aligned}$$

NOw To find the length.

1.5 Steps

$$V_2 = L\ A$$

Hence

$$L = \frac{V_2}{A}$$

But $\frac{P_1 V_1}{T_1} = mR$ and $\frac{P_2 V_2}{T_2} = mR$, so $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$, hence $\boxed{V_2 = \frac{P_1 T_2}{P_2 T_1} V_1}$
So

$$L = \frac{V_2}{A} = \frac{1}{A}\left(\frac{P_1 T_2}{P_2 T_1} V_1\right)$$

but $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ hence

$$\begin{aligned} L &= \frac{1}{A}\left(\frac{P_1 T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}{P_2 T_1} V_1\right) \\ &= \frac{1}{A}\left(\frac{P_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}{P_2} V_1\right) \end{aligned}$$

1.5.1 Numerical

Given $A = 5cm^2$, $V_1 = 10cm^3$ so

$$\begin{aligned} L &= \frac{1}{5}\left(\frac{(15 \times 10^3)\left(\frac{200}{15 \times 10^3}\right)^{\frac{1.4-1}{1.4}}}{200} 10\right) \\ &= 43.688 \text{ cm} \end{aligned}$$