

13.5 (a) At  $t = 0$ ,  $x = 0$  and  $v$  is positive (to the right). Therefore, this situation corresponds to

$$x = A \sin \omega t \quad \text{and} \quad v = v_i \cos \omega t$$

Since  $f = 1.50 \text{ Hz}$ ,  $\omega = 2\pi f = 3.00\pi$

Also,  $A = 2.00 \text{ cm}$ , so that  $x = (2.00 \text{ cm}) \sin 3.00\pi t$

(b)  $v_{\max} = v_i = A\omega = (2.00)(3.00\pi) = 6.00\pi \text{ cm/s}$

The particle has this speed at  $t = 0$  and next at  $t = \frac{T}{2} = \frac{1}{3} \text{ s}$

(c)  $a_{\max} = A\omega^2 = 2(3.00\pi)^2 = 18.0\pi^2 \text{ cm/s}^2$

The acceleration has this positive value for the first time at

$$t = \frac{3T}{4} = 0.500 \text{ s}$$

(d) Since  $T = \frac{2}{3} \text{ s}$  and  $A = 2.00 \text{ cm}$ , the particle will travel  $8.00 \text{ cm}$  in this time.

Hence, in  $1.00 \text{ s} \left( = \frac{3T}{2} \right)$ , the particle will travel

$$8.00 \text{ cm} + 4.00 \text{ cm} = 12.0 \text{ cm}$$

13.7  $k = \frac{F}{x} = \frac{(10.0 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{3.90 \times 10^{-2} \text{ m}} = 2.51 \text{ N/m}$  and

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{25.0 \times 10^{-3} \text{ kg}}{2.51 \text{ N/m}}} = 0.627 \text{ s}$$

13.10  $m = 1.00 \text{ kg}$ ,  $k = 25.0 \text{ N/m}$ , and  $A = 3.00 \text{ cm}$

At  $t = 0$ ,  $x = -3.00 \text{ cm}$

(a)  $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0}{1.00}} = 5.00 \text{ rad/s}$

so that,  $T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = 1.26 \text{ s}$

(b)  $v_{\max} = A\omega = (3.00 \times 10^{-2} \text{ m})(5.00 \text{ rad/s}) = 0.150 \text{ m/s}$

$$a_{\max} = A\omega^2 = (3.00 \times 10^{-2} \text{ m})(5.00 \text{ rad/s})^2 = 0.750 \text{ m/s}^2$$

(c) Because  $x = -3.00 \text{ cm}$  and  $v = 0$  at  $t = 0$ , the required solution is

$$x = -A \cos \omega t$$

or  $x = -3.00 \cos (5.00t) \text{ cm}$

$$v = \frac{dx}{dt} = 15.0 \sin (5.00t) \text{ cm/s}$$

$$a = \frac{dv}{dt} = 75.0 \cos (5.00t) \text{ cm/s}^2$$

13.17 By conservation of energy,  $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$$v = \sqrt{\frac{k}{m}} x = \sqrt{\frac{5.00 \times 10^6}{10^3}} (3.16 \times 10^{-2} \text{ m}) = \boxed{2.23 \text{ m/s}}$$

13.22 (a)  $E = \frac{1}{2}kA^2$ , so if  $A' = 2A$ ,  $E' = \frac{1}{2}k(A')^2 = \frac{1}{2}k(2A)^2 = 4E$

Therefore  $E$  increases by factor of 4.

(b)  $v_{\max} = \sqrt{\frac{k}{m}} A$ , so if  $A$  is doubled,  $v_{\max}$  is doubled.

(c)  $a_{\max} = \frac{k}{m} A$ , so if  $A$  is doubled,  $a_{\max}$  also doubles.

(d)  $T = 2\pi\sqrt{\frac{m}{k}}$  is independent of  $A$ , so the period is unchanged.

\*13.27 The swinging box is a physical pendulum with period  $T = 2\pi\sqrt{\frac{I}{mgd}}$ .

The moment of inertia is given approximately by

$$I = \frac{1}{3}mL^2 \text{ (treating the box as a rod suspended from one end).}$$

Then, with  $L = 1.0 \text{ m}$  and  $d \approx L/2$ ,

$$T = 2\pi\sqrt{\frac{(1/3)mL^2}{mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.0 \text{ m})}{3(9.8 \text{ m/s}^2)}} = 1.6 \text{ s} \quad \text{or} \quad T \sim \boxed{10^0 \text{ s}}$$

13.49 Assume that each spring supports an equal portion of the car's mass, i.e.  $\frac{m}{4}$ .

$$\text{Then } T = 2\pi\sqrt{\frac{m}{4k}} \quad \text{and} \quad k = \frac{4\pi^2 m}{4T^2} = \frac{4\pi^2 1500}{(4)(1.50)^2} = \boxed{6580 \text{ N/m}}$$