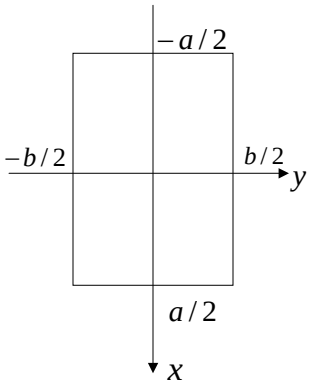
Torsion of Rectangular Bars



$$\nabla^2\phi = -2G\alpha$$

$$\phi\big|_\Gamma = 0$$

Series Solution:

$$\phi = G\alpha\left(\frac{b^2}{4} - y^2\right) - \frac{8G\alpha b^2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}\frac{\cosh\lambda_n x}{\cosh(\lambda_n a/2)}\cos\lambda_n y$$

where $\lambda_n = (2n+1)\dfrac{\pi}{b}$

Finite Element Program

MATLAB program which solves the 2D Poisson equation with Dirichlet boundary conditions:

$$\nabla^2\phi + f(x,y) = 0$$

$$\phi\big|_\Gamma = 0$$

http://www.csit.fsu.edu/~burkardt/m_src/fem2d_poisson/fem2d_poisson.html

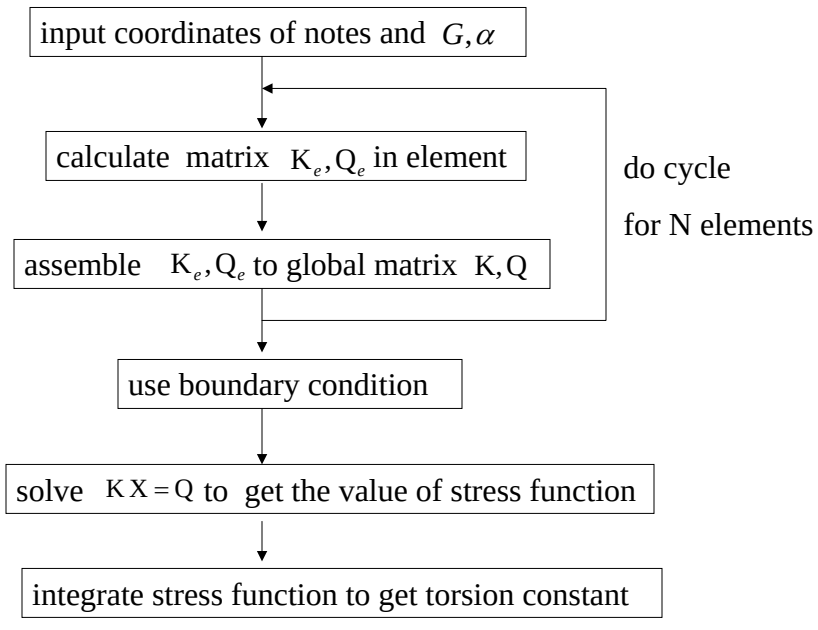
Notice: There is a minor error in “nodes_plot.m”

How to use this program?

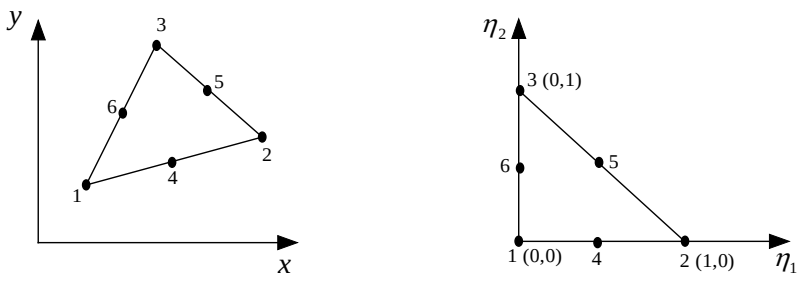
"rhs.m": modify the function $f(x, y)$

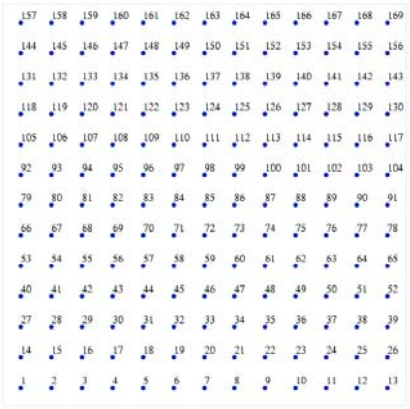
"exact.m": input exact solution if it exists

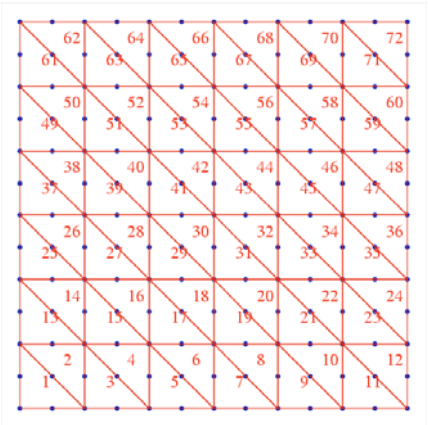
Run "fem2d_poisson.m" , get the results!



Quadrate Triangle Element





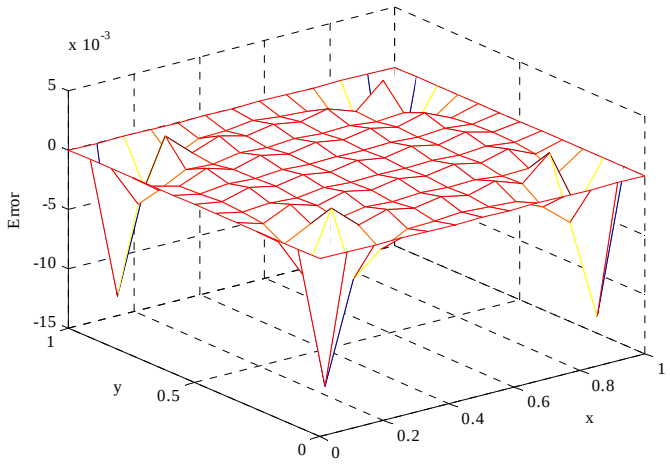


$a = b = 1mm$

$\alpha = 0.001$

$G = 10^8 MPa$

Error of the value of stress function



$$error = \frac{\phi_{series} - \phi_{fem}}{\phi_{series}}$$

Larger error in corner nodes?

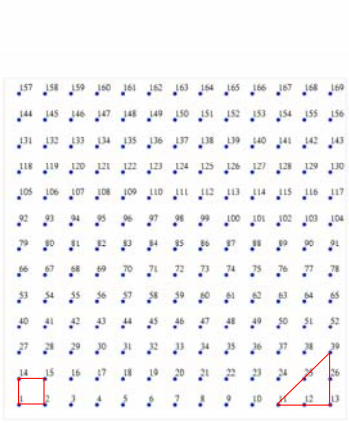
Torsion Constant

$$D_t = \frac{M_t}{\alpha} = \frac{2\iint_S \phi dxdy}{\alpha}$$

$D_t^{series} = 1.4058e+007$

$D_t^{fem} = 1.3828e+007$

$error = 1.64\%$



Notice: we do the integration in rectangular element, but it is better to do it in triangular element.