Finite Volume Method (FVM)

■ Notes

```
N := a + b ξ + c η + d ξ η + e ξ^2 + f η^2 + g ξ^2 η + h ξ η^2 + i ξ^2 η^2;
x := x1 + (x2 - x1) ξ + (x3 - x1) η; y := y1 + (y2 - y1) ξ + (y3 - y1) η; J = {{∂ξ x, ∂η x}, {∂ξ y, ∂η y}};
Inverse[J].{∂ξ N, ∂η N} // MatrixForm // FullSimplify
Clear[N]; N[ξ_, η_] := a + b ξ + c η + d ξ η + e ξ^2 + f η^2 + g ξ^2 η + h ξ η^2 + i ξ^2 η^2;
dNdx[ξ_, η_] :=
  ((-y1 + y3) (b + η (d + h η) + 2 (e + η (g + i η)) ξ) + (x1 - x3) (c + 2 f η + ξ (d + g ξ + 2 η (h + i ξ))))/
   (x3 (y1 - y2) + x1 (y2 - y3) + x2 (-y1 + y3));
dNdy[ξ_, η_] :=
  ((y1 - y2) (b + η (d + h η) + 2 (e + η (g + i η)) ξ) + (-x1 + x2) (c + 2 f η + ξ (d + g ξ + 2 η (h + i ξ))))/
   (x3 (y1 - y2) + x1 (y2 - y3) + x2 (-y1 + y3));
Solve[{N[0, 0] == q1, dNdx[0, 0] == q4, dNdy[0, 0] == q7, N[1, 0] == q2, dNdx[1, 0] == q5,
  dNdy[1, 0] == q8, N[0, 1] == q3, dNdx[0, 1] == q6, dNdy[0, 1] == q9}, {a, b, c, d, e, f, g, h, i}]
{}
```

$$N = a + b\,\xi + c\,\eta + d\,\xi\,\eta + e\,\xi^2 + f\,\eta^2 + g\,\xi^2\,\eta + h\,\xi\,\eta^2 + i\,\xi^2\,\eta^2$$

$$a = q_1$$

$$b = (-x1 + x2)\,q_4 + (-x1 + x3)\,q_7$$

$$c = (-y1 + y2)\,q_4 + (-y1 + y3)\,q_7$$

$$d = d$$

$$e = -q_1 + q_2 + (x1 - x2)\,q_4 + (x1 - x3)\,q_7$$

$$f = -q_1 + q_3 + (y1 - y2)\,q_4 + (y1 - y3)\,q_7$$

$$g = -d + \frac{1}{x1 - x3}\Big(-2\,(y1 - y3)\,q_1 + 2\,(y1 - y3)\,q_2 - (x2\,y1 + x3\,y1 - x3\,y2 - x2\,y3 + x1\,(-2\,y1 + y2 + y3))\,q_4 + (x3\,y1 + x1\,y2 - x3\,y2 - x1\,y3 + x2\,(-y1 + y3))\,q_5\Big) + 2\,(y1 - y3)\,q_7$$

$$h = -d + \frac{1}{y1 - y3}\Big(-2\,(x1 - x3)\,q_1 + 2\,(x1 - x3)\,q_3 - (x2\,y1 + x3\,y1 - x3\,y2 - x2\,y3 + x1\,(-2\,y1 + y2 + y3))\,q_4 + (-x3\,y1 - x1\,y2 + x3\,y2 + x2\,(y1 - y3) + x1\,y3)\,q_6\Big) + 2\,(x1 - x3)\,q_7$$

$$i = i$$

---

pg. 805

shape functions : $N_1 = \xi 3 = 1 - \xi - \eta$, $N_2 = \xi 1 = \xi$, $N_3 = \xi 2 = \eta$

```
u := a ξ1 + b ξ2 + c ξ3 + d ξ1 ξ2 + e ξ2 ξ3 + f ξ3 ξ1 +
   g (ξ1^2 ξ2 + ξ1 ξ2 ξ3 (3 (1 - μ3) ξ1 - (1 + 3 μ3) ξ2 + (1 + 3 μ3) ξ3) / 2) +
   h (ξ2^2 ξ3 + ξ1 ξ2 ξ3 (3 (1 - μ1) ξ2 - (1 + 3 μ1) ξ3 + (1 + 3 μ1) ξ1) / 2) +
   i (ξ3^3 ξ1 + ξ1 ξ2 ξ3 (3 (1 - μ2) ξ3 - (1 + 3 μ2) ξ1 + (1 + 3 μ2) ξ2) / 2);
μ1 = (ℓk^2 - ℓj^2) / ℓi^2
```

---

see Patankar, 1985
Malagasekara

- **http://www.amazon.com/gp/offer-listing/0582218845/sr=8-1/qid=1157653744/ref=sr_1_1/002-2175139-6340059?ie=UTF8&s=books**
  zdhan@care.eng.uci.edu