MAE 171 **Digital Control Systems** A. Sideris

Homework # 7

Due: June 10, 2005

Problem 1

Consider the control system shown in Fig. 1. Plot the root loci as the gain K is varied from 0 to ∞. Determine the critical value of gain K for stability. The sampling period is $T = 0.1 secs$. What value of gain K will yield a damping ration ζ of the closed-loop poles equal to 0.5? With gain K set to yield $\zeta = 0.5$, determine the damped natural frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and the number of samples per cycle of damped sinusoidal oscillation.

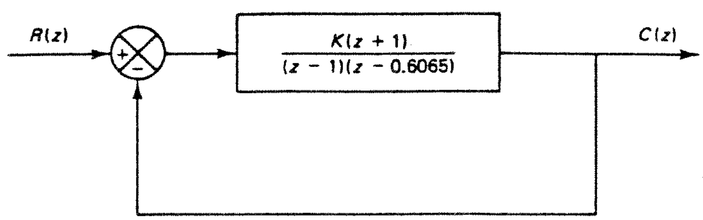

Figure 1:

Problem 2

Referring to the digital control system in Fig. 2, design a *lead type* digital controller $G_D(z) = K_D\frac{z-z_0}{z-z_p}$ such as the damping ratio ζ of the dominant closed loop poles is 0.5 and the number of samples per cycle of damped sinusoidal oscillation is 8. Assume that the sampling period is $T = 0.1 secs$. Determine the static velocity error constant. Then add a *lag type* section in the controller $\frac{z-\bar{z}_0}{z-\bar{z}_p}$ to boost the velocity error constant by a factor or 2. (Take the zero of the lag section at $\bar{z}_0 = 0.99$.) Simulate the unit step response of the discrete-time system that represents the sampled-data system at the sampling instances using MATLAB (sampled-data system simulation is optional.)

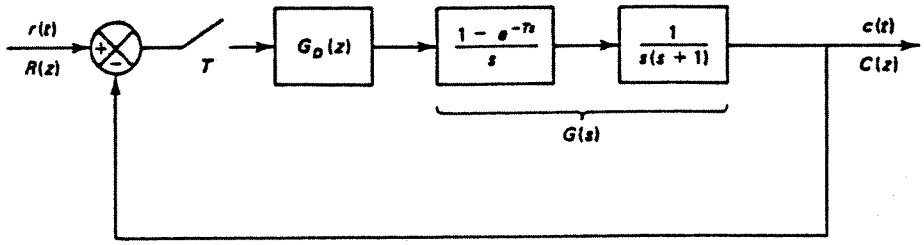

Figure 2:

Problem 3—(Problem 11-1 in the text—modified)

Given $G(s) = \frac{s+2}{s^2+4s+3}$ and $T = 1secs$, find equivalent discrete-time systems using the

1. standard z-transform (impulse invariance)
2. step invariance
3. backward difference
4. forward difference
5. bilinear z-transform
6. matched z-transform