

Homework #3

Solutions

$$3-1, (a) E^*(s) = \sum_{n=0}^{\infty} e(nT) e^{-nTs} \quad (b) E(z) = \sum_{n=0}^{\infty} e(nT) z^{-n}$$

$$(c) E^*(s) = E(z) \Big|_{z=e^{sT}}$$

$$3-4 (e) E(s) = \frac{s+2}{s(s+1)} \rightarrow e(t) = \mathcal{L}^{-1} \left[\frac{s+2}{s(s+1)} \right] = \mathcal{L}^{-1} \left[\frac{2}{s} - \frac{1}{s+1} \right] \rightarrow \\ e(t) = (2 - e^{-t}) u(t) \rightarrow e(kT) = (2 - e^{-kT}) u(kT) \rightarrow$$

$$E(z) = \bar{z} [e(kT)] = \bar{z} [2 u(kT) - e^{-kT} u(kT)] = \frac{2\bar{z}}{z-1} - \frac{\bar{z}}{z-e^{-T}} \Rightarrow$$

$$\boxed{E(z) = \frac{z(z+1-2e^{-T})}{(z-1)(z-e^{-T})}} \xrightarrow{z=e^{sT}} \boxed{E^*(s) = \frac{e^{sT}(e^{sT}+1-2e^{-T})}{(e^{sT}-1)(e^{sT}-e^{-T})}}$$

$$(f) E(s) = \frac{2}{s^2 + 2s + 5} = \frac{2}{(s+1)^2 + 2^2} \xrightarrow[\# 16]{\text{Tables}} (a=1, \omega=2)$$

$$\boxed{E(z) = \frac{z^{-T} \sin 2T}{z^2 - 2(e^{-T} \cos 2T)z + e^{-2T}}} \xrightarrow{z=e^{sT}} \boxed{E^*(s) = \frac{e^{sT-T} \sin 2T}{e^{2sT} - 2(e^{-T} \cos 2T)e^{sT} + e^{-2T}}}$$

$$3-15. (a) 4, 7 \text{ rad/s}$$

$$(b) \quad \begin{array}{r} 4 \\ 22 \cancel{2} 4 = \underline{18}, 24 \\ 44 \cancel{2} 4 = \underline{40}, 48 \end{array} \quad \begin{array}{r} 7 \\ 22 \cancel{2} 7 = \underline{15}, 29 \\ 44 \cancel{2} 7 = \underline{37} \end{array}$$

(c) Same as (b)

(d) Same as (b)

$$3.16 \text{ (a)} \quad e(t) = 4 \sin 7t \longrightarrow E(j\omega) = j4\pi [\delta(\omega+7) - \delta(\omega-7)]$$

Frequencies in $e^*(t)$: $\pm 7 \pm n\omega_s$, $\omega_s = 4 \frac{\text{rad}}{\text{sec}}$, $n=0, 1, 2, 3, \dots$

$\therefore n=0, \pm 7 \text{ rad/sec}$

$n=1, \pm 11, \pm 3$

$n=2, \pm 1, \pm 15, \text{etc.}$

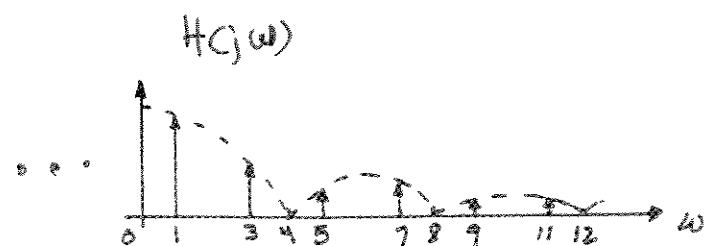
$\left. \begin{array}{l} \\ \end{array} \right\} \omega = 1 \text{ rad/sec is frequency component of output with largest amplitude.}$

(b) Let $E_1^*(j\omega)$ be the part of $E^*(j\omega)$ that comes from the frequency component at $\omega = 1 \text{ rad/sec}$:

$$\text{Then } E_1^*(j\omega) = j \frac{4\pi}{7} [\delta(\omega-1) - \delta(\omega+1)]$$

NOTE the minus sign in $E_1^*(j\omega)$! This is because the impulse at $\omega=1(-1)$ in $E_1^*(j\omega)$ comes from the impulse at $\omega=-1(1)$ in $E(j\omega)$.

Then with $H_1(j\omega)$ the part of the reconstructed signal that comes from the frequency component at $\omega = 1 \text{ rad/sec}$ in $E^*(j\omega)$, we have: $H_1(j\omega) = (h_{10}(j\omega)) E_1^*(j\omega)$



The ZOH frequency response is:

$$G_{ho}(j\omega) = T \sin\left(\frac{\pi\omega}{\omega_s}\right) \cdot e^{-j\frac{\pi\omega}{\omega_s}} \sim$$

$$G_{ho}(j1) = T \frac{\sin \frac{\pi}{4}}{\frac{\pi}{4}} e^{-j\frac{\pi}{4}} = \frac{T2\sqrt{2}}{\pi} e^{-j\frac{\pi}{4}}, \text{ and}$$

$$G_{ho}(-j1) = T \frac{\sin\left(-\frac{\pi}{4}\right)}{\left(-\frac{\pi}{4}\right)} e^{j\frac{\pi}{4}} = \frac{T2\sqrt{2}}{\pi} e^{j\frac{\pi}{4}}.$$

$$\text{Therefore, } H_1(j\omega) = \frac{T2\sqrt{2}}{\pi} \cdot \left(+j\frac{4\pi}{1}\right) \left[e^{-j\frac{\pi}{4}}\delta(\omega-1) - e^{j\frac{\pi}{4}}\delta(\omega+1) \right] \sim$$

$$H_1(j\omega) = \frac{8\sqrt{2}}{\pi} (-j\pi) \left[e^{j\frac{\pi}{4}}\delta(\omega+1) - e^{-j\frac{\pi}{4}}\delta(\omega-1) \right] \sim$$

$$\sim \boxed{h_1(t) = \frac{8\sqrt{2}}{\pi} \sin(t - \frac{\pi}{4})}$$

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(c) t=linspace(0,4*pi,200);
plot(t,4*sin(7*t),'--',t,-8*sqrt(2)/pi*sin(t-pi/4))
hold on
t1=(0:2*pi/4:4*pi);
h=4*sin(7*t1);
for i=1:length(t1)-1,
    f=plot([t1(i) t1(i+1)],[h(i) h(i)],'LineWidth',3)
    f=plot([t1(i+1) t1(i+1)],[h(i) h(i+1)],'LineWidth',3)
end
hold off
grid
title('e(t)--dashed, h_1(t)--solid, h(t)--heavy solid')
xlabel('time(secs)')
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see plot on next page.

$$(d) \text{ ratio} = \frac{|G_{ho}(j1)|}{|G_{ho}(j7)|} = \left| \frac{\sin\left(\frac{\pi}{4}\right)}{\sin\left(\frac{7\pi}{4}\right)} \right| = \left| \frac{8 \cdot \sin\left(\frac{\pi}{4}\right)}{\sin\left(\frac{7\pi}{4}\right)} \right| = 7$$

