Homework #2

SOLUTIONS

2-10. $x(k) - 3x(k-1) + 2x(k-2) = e(k)$, $e(k) = \begin{cases} 1, & k = 0, 1 \\ 0, & k \geq 2 \end{cases}$

(a) $x(0) = e(0) = 1$

$x(1) = e(1) + 3x(0) = 4$

$x(2) = e(2) + 3x(1) - 2x(0) = 10$

$x(3) = 0 + 3(10) - 2(4) = 22$

$x(4) = 0 + 3(22) - 2(10) = 46$

(b) $[1 - 3z^{-1} + 2z^{-2}]X(z) = E(z) = 1 + z^{-1} = \frac{z+1}{z}$

$X(z) = \frac{z^2}{(z-1)(z-2)} \times \frac{z+1}{z} = \frac{z(z+1)}{(z-1)(z-2)} = z\left[\frac{-2}{z-1} + \frac{3}{z-2}\right]$

$\therefore x(k) = \underline{-2 + 3(2)^k}$, $k \geq 0$

(c) <u>No</u>, since the final value does not exist.

2-14. (a) $y(k+1) = y(k) + T\,\frac{x(k) + x(k+1)}{2}$

(b) $zY(z) = Y(z) + \frac{T}{2}[X(z) + zX(z)] \Rightarrow \underline{Y(z) = \frac{T}{2}\,\frac{z+1}{z-1}X(z)}$

2-12.(a) $[1 - z^{-1} + z^{-2}]X(z) = E(z) = \frac{z}{z-1}$

$X(z) = \frac{z^3}{(z-1)(z^2-z+1)}$, poles: $z = \frac{1}{2} \pm j\frac{\sqrt{3}}{2} = 1\angle \pm 60^\circ$

$\frac{X(z)}{z} = \frac{1}{z-1} + \frac{k_1}{z-p_1} + \frac{k_1^*}{z-p_1^*}$ with $p = 1\angle 60^\circ$

$k_1 = \frac{z^2}{(z-1)(z-1\angle 60^\circ)}\Big|_{z=1\angle 60^\circ} = \frac{1\angle 120^\circ}{(.5+j.866-1)(.5+j.866-.5+j.866)}$

$= \frac{1\angle 120^\circ}{1\angle 120^\circ [j2(0.866)]} = 0.5774\angle -90^\circ$

$\therefore aT = \ln |p_1| = \underline{0}$; $bT = \arg p_1 = \underline{\frac{\pi}{3}}$

$A = 2|k_1| = \underline{1.155}$; $\theta = \arg k_1 = \underline{-90^\circ}$

$\therefore x(k) = 1 + 1.155 \cos(\frac{\pi}{3}k - 90^\circ) = 1 + 1.155 \sin(\frac{\pi}{3}k)$, $K \geq 0$

$\therefore x(0) = \underline{1}$, $x(1) = \underline{2}$, $x(2) = \underline{2}$

(b) $z^3 - 2z^2 + 2z - 1 \overline{)\, z^3}$ $\quad 1 + 2z^{-1} + 2z^{-2} + \cdots$

$z^3 - 2z^2 + 2z - 1$

$2z^2 - 2z + 1$

$2z^2 - 4z + 4 - 2z^{-1}$

$2z + \cdots$

$\therefore x(0) = \underline{1}$

$x(1) = \underline{2}$

$x(2) = \underline{2}$

(c) $x(k) = 1 + x(k-1) - x(k-2)$

$x(0) = 1 + 0 - 0 = \underline{1}$

$x(1) = 1 + 1 - 0 = \underline{2}$

$x(2) = 1 + 2 - 1 = \underline{2}$

(d) No, 3 poles for $X(z)$ on the unit circle.

2-12 (a) Alternative solution using formula given in class.

$$\frac{X(z)}{z} = \frac{A}{z-1} + \frac{Bz+C}{z^2-z+1}$$

$$A = \left[(z-1)\frac{X(z)}{z}\right]\Big|_{z=1} = \frac{z^3}{z^2-z+1}\Big|_{z=1} = 1$$

$$z=0 \rightsquigarrow \quad 0 = \frac{1}{0-1} + \frac{C}{0-0+1} \rightarrow C=1$$

$$z=-1 \rightarrow \frac{-1}{3(-2)} = \frac{1}{-2} + \frac{B(-1)+1}{3} \rightsquigarrow -1 = 3 + (B-1)\cdot 2 \rightarrow B=0$$

$$\therefore \boxed{X(z) = \frac{z}{z-1} + \frac{z}{z^2-z+1}}$$

$$\mathcal{Z}\left[\frac{z}{z-1}\right] = u(k) \; ; \quad \mathcal{Z}\left[\frac{z}{z^2-z+1}\right] = a^k\left[c_1 \sin bk + c_2 \cos bk\right], \; k \geq 0$$

Formulas: $\left\{\begin{matrix} b_1 \equiv B = 0, & b_2 \equiv C = 1 \\ a_1 = -1, & a_2 = 1 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} a = \sqrt{a_2} = 1, & b = \arccos\left[\frac{1}{2}\right] = \pi/3 \\ c_1 = \frac{2b_2 - a_1 b_1}{\sqrt{4a_2 - a_1^2}} = \frac{2}{\sqrt{3}}, & c_2 = b_1 = 0 \end{matrix}\right\}$

$$\therefore \boxed{x(k) = u(k) + (1)^k \frac{2}{\sqrt{3}} \sin\left(\frac{\pi k}{3}\right) u(k)}$$