

given  
gray  
levels  
also

$$\begin{aligned} \beta &= 10C_1 + 2C_2 + 20C_3 + C_4 \\ \gamma &= 10C_1 + 6C_2 + 60C_3 + C_4 \\ 1 &= 12C_1 + 2C_2 + 24C_3 + C_4 \\ 12 &= 12C_1 + 6C_2 + 72C_3 + C_4 \end{aligned}$$

$$r(x, y) = \underbrace{C_1 x + C_2 y}_{r(x, y)} + \underbrace{C_3 xy + C_4}_{s(x, y)}$$

$$s(x, y) = \underbrace{C_5 x + C_6 y}_{r(x, y)} + \underbrace{C_7 xy + C_8}_{s(x, y)}$$

To obtain find gray level, use interpolation  $g(x, y) = ax + by + cxy + d$  for 4 corners near  $r(x, y), s(x, y)$

$$\text{VAR}[K f(x, y)] = K^2 \text{VAR}[f(x, y)] \quad \text{VAR}[n_1 + n_2] = \sum \text{VAR}(n_i) \text{ if } n_i \text{ are independent}$$

Median Filter: reduce noise with less blurring than Linear smoothing filters. good for impulse noise. salt/pepper

Sharpening filter: has high response for areas of image with large gradient. low response for constant/low change

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}, \text{ discrete version: } \nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

$$\frac{\partial f}{\partial x} = f(x+1) - f(x) \quad \frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

$$\text{DFT: } F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi \left(\frac{ux}{M} + \frac{vy}{N}\right)}, \text{ IDFT: } f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi \left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$\boxed{\text{Gaussian Filter}}: H(u, v) = A e^{-\frac{D^2}{2D_o^2}} \Rightarrow h(x, y) = A 2\pi D_o^2 e^{-2\pi^2 D_o^2 D^2}, \text{ Larger } D \text{ in freq.} \Rightarrow \text{Smaller in space. Gaussian can't eliminate noise. since } \neq 0.$$

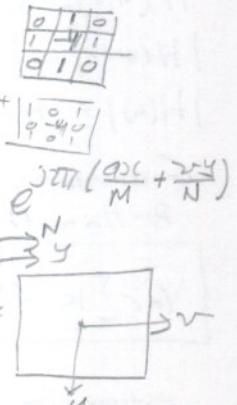
Ripples in Sinc  $\rightarrow$  ringing. wide filter in freq  $\rightarrow$  narrow in spec

$$\text{Butterworth Low pass: } H(u, v) = \frac{1}{1 + \left(\frac{D}{D_o}\right)^{2n}}, \text{ high pass: } \frac{1}{1 + \left(\frac{D_o}{D}\right)^{2n}}$$

$$\text{Gaussian PDF: } p(z) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

$$M = \frac{1}{N} \sum f(x, y) \quad \sigma^2 = \frac{1}{N} \sum (f(x, y) - M)^2 \quad \text{Variation.}$$

$$\text{Gaussian Noise: } p(z) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$



Given  $g_1, g_2$ , where  $g_1 = f + n_1$ ,  $g_2 = f + n_2 \Rightarrow \sigma_g^2 = 2\sigma_n^2$ . So to find  $\sigma_g$ , find  $\sigma_n$ .

$$\text{AMF: } \frac{1}{MN} \sum g(s, t) : \text{Linear - good for pepper noise.}$$

$$\text{CMF: } \frac{1}{\pi} g(s, t) \frac{1}{MN} : \text{Nonlinear - darkness more than AMF. more randomness in gray level, darkness. Good for Salt noise. darker noise down more. GMF, better edge blur, smoother.}$$

$$\text{HMF: } \frac{MN}{\sum g(s, t)} : \text{Nonlinear: unstable for small gray level change. darkness more than AMF. good for salt, bad for pepper.}$$

$$\text{median: good for salt/pepper. bad for edges.} \quad \boxed{\text{Contraharmonic CMF}} \quad \frac{\sum g(s, t)^{Q+1}}{\sum g(s, t)^Q} \quad Q=0 \rightarrow \text{AMF}, Q>0 \text{ smoothing regions with gray level variability.} \quad Q=-1 \rightarrow \text{HMF}$$

bad for salt noise.  $Q=1$ , good for pepper.  $\triangleright Q_{\text{dark}}, +Q_{\text{bright}}$ .

Adaptive noise filtering: smooth noise but not edges. Adaptive median filter: remove impulse noise median filters: good for impulse noise

due to noise  
Restoration by frequency

$$H(u, v) = \begin{cases} 1, & D \leq D_o - \frac{w}{2} \\ 1, & D \geq D_o + \frac{w}{2} \\ 0, & \text{else} \end{cases}$$



Band Reject

IF median NOT impulse then  
else IF pixel is salt then  $f = \text{pixel}$  else  $f = \text{median}$

increase window size, try again

$$-\frac{1}{2} \left( \frac{D^2 - D_o^2}{DW} \right)^2$$

Fourier Transfer of  
impulse is constant

$$\boxed{\text{Butterworth: }} H(u, v) = \frac{1}{1 + \left(\frac{D}{D_o}\right)^{2n}}, \text{ Gaussian: } H(u, v) = 1 - e^{-\frac{1}{2} \left( \frac{D^2 - D_o^2}{D_o^2} \right)}$$

$$\boxed{\text{Notch Filter: }} H(u, v) = \frac{1}{1 + \left(\frac{D_o^2}{D_1 D_2}\right)^n}, \text{ Gaussian: } 1 - e^{-\frac{1}{2} \left( \frac{D^2 - D_o^2}{D_o^2} \right)}$$

$$S(x,y) \Leftrightarrow 1 \\ A 2\pi \sigma^2 e^{-2\pi^2 \sigma^2 (x^2 + y^2)} \Leftrightarrow Ae^{-\frac{(u^2+v^2)}{2\sigma^2}}$$

$$\text{Convolution } f(x,y) * h(x,y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) h(x-m, y-n)$$

$$f(x-x_0, y-y_0) \Leftrightarrow F(u,v) e^{-j2\pi \left( \frac{u x_0}{M} + \frac{v y_0}{N} \right)}$$

$$f(x,y) (-1)^{x+y} \Leftrightarrow F(u-\frac{M}{2}, v-\frac{N}{2}), \quad f(ax, by) \Leftrightarrow \frac{1}{|ab|} F(\frac{u}{a}, \frac{v}{b})$$

$$H(u,v) = H^*(-u,-v)$$

$$|H(u,v)| = |H(-u,-v)|$$

$$|H(u)| = |H(-u)|$$

Gaussian: Norringing.

Br-Hannuth oder 2: more smooth, but can produce ringing

$$\boxed{\text{Var}[K f(x,y)] = K^2 \text{Var}[f(x,y)]} \quad \boxed{\text{Var}[\Sigma] = \sum \text{Var}(e_i) \text{ if linearly independent}}$$

$$\text{estimation by filtering } H(u,v) = e^{-K(u^2+v^2)^{5/6}}$$

Band Reject filters

$$H(u,v) = \frac{1}{1 + \left( \frac{D \omega}{D^2 - D_0^2} \right)^2} \quad \leftarrow \text{Butterworth}$$

$$H(u,v) = 1 - e^{-\frac{1}{2} \left( \frac{D^2 - D_0^2}{D \omega} \right)^2} \quad \leftarrow \text{Gaussian}$$

$$\text{mean } u = \sum z_i p(z_i)$$

$$\sigma^2 = \sum (z_i - u)^2 p(z_i)$$

} estimation of noise parameters

