HW 6, EECS 203A
Problem 5.13, Digital Image Processing, 2nd edition by Gonzalez, Woods.
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**Question**

Obtain equations for the bandpass filters corresponding to the bandreject filters in eqs 5.4-1 through 5.4-3

**Solution**

A bandpass filter can be obtained from a bandreject filter by the following relation

$$H_{bandpass}(u,v) = 1 - H_{bandreject}(u,v)$$

eqs 5.4-1 through 5.4-3 give the bandpass reject equations, hence to obtain the bandpass equations we need to substitue in the above.

**ideal bandpass**

Here, every where the bandreject has a value of 1, we make it zero, and every where it is 0 we make it 1. Ideal bandreject is given by

$$\mathbf{H}_{br}(u,v) = \begin{cases} 1 & \textbf{if } \mathbf{D}(u,v) < \mathbf{D}_0 - \frac{W}{2} \\ 1 & \textbf{if } \mathbf{D}(u,v) > \mathbf{D}_0 + \frac{W}{2} \\ 0 & \textbf{otherwise} \end{cases}$$

Hence an ideal bandpass is given by

$$\mathbf{H}_{bp}(u,v) = \begin{cases} 0 & \textbf{if } \mathbf{D}(u,v) < \mathbf{D}_0 - \frac{W}{2} \\ 0 & \textbf{if } \mathbf{D}(u,v) > \mathbf{D}_0 + \frac{W}{2} \\ 1 & \textbf{otherwise} \end{cases}$$

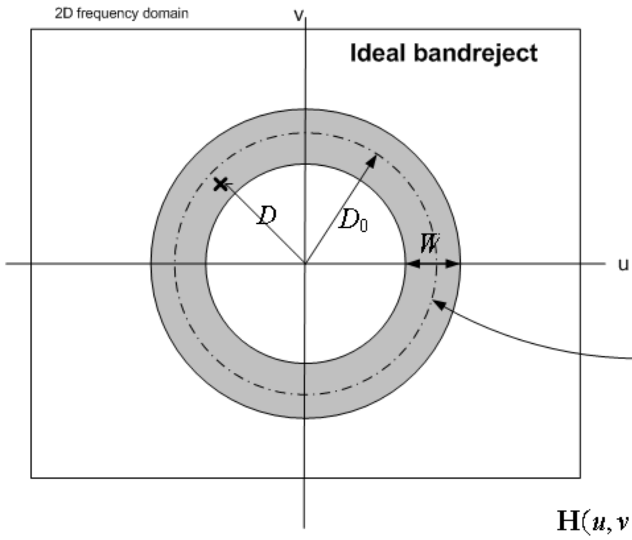
This is illustrated in the following diagram on next page

**bandpass Butterworth filter**

The Butterworth reject filter of order $n$ is given by

$$H_{br}(u,v) = \frac{1}{1 + \left(\frac{D\ W}{D^2 - D_0^2}\right)^{2n}}$$

Where $D$ is the distance from center of 2D spectrum to any point. $W$ is the width of the band, $D_0$ is the distance from center of spectrum to the center of the band. (note: $D$ and $D_0$ should be written as $D(u,v)$, $D_0(u,v)$, but for clarity of expression, I did not add these).
Hence, the butterworth bandpass filter is



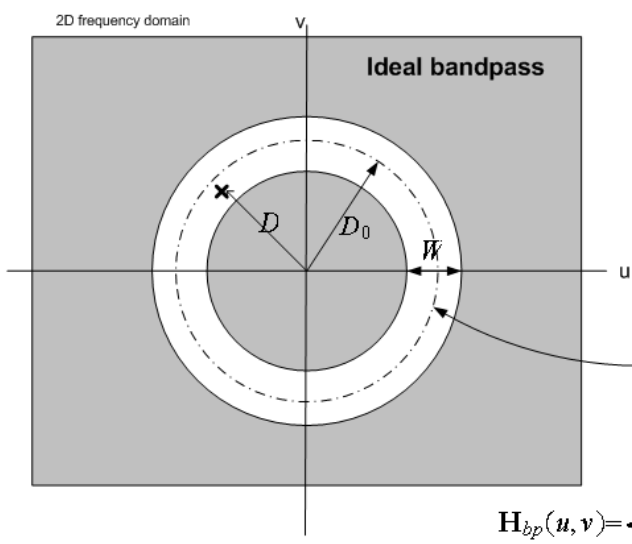

$D$ Is the distance from the origin to any point (u,v) in the 2D spectrum space

$D_0$ Is the distance from the origin to the center of the reject ring

$W_0$ Is the width of the bandreject ring

Any frequency inside this range, will be rejected

$$H(u,v)=\begin{cases} 1 & \textbf{if } D(u,v) < D_0 - \frac{W}{2} \\ 1 & \textbf{if } D(u,v) > D_0 + \frac{W}{2} \\ 0 & \textbf{otherwise} \end{cases}$$




$D$ Is the distance from the origin to any point (u,v) in the 2D spectrum space

$D_0$ Is the distance from the origin to the center of the reject ring

$W_0$ Is the width of the bandreject ring

Any frequency inside this range, will be passed

$$H_{bp}(u,v)=\begin{cases} 0 & \textbf{if } D(u,v) < D_0 - \frac{W}{2} \\ 0 & \textbf{if } D(u,v) > D_0 + \frac{W}{2} \\ 1 & \textbf{otherwise} \end{cases}$$

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*Nov 20, 2004*

$$
\begin{aligned}
H_{bp}(u,v) &= 1-\frac{1}{1+\left[\frac{D\ W}{D^2-D_0^2}\right]^{2n}} \\
&= \frac{\left[\frac{D\ W}{D^2-D_0^2}\right]^{2n}}{1+\left[\frac{D\ W}{D^2-D_0^2}\right]^{2n}} = \frac{\frac{(D\ W)^{2n}}{\left(D^2-D_0^2\right)^{2n}}}{1+\frac{(D\ W)^{2n}}{\left(D^2-D_0^2\right)^{2n}}} = \frac{\frac{(D\ W)^{2n}}{\left(D^2-D_0^2\right)^{2n}}}{\frac{\left(D^2-D_0^2\right)^{2n}+(D\ W)^{2n}}{\left(D^2-D_0^2\right)^{2n}}} \\
&= \frac{(D\ W)^{2n}}{\left(D^2-D_0^2\right)^{2n}+(D\ W)^{2n}} = \frac{1}{\frac{\left(D^2-D_0^2\right)^{2n}+(D\ W)^{2n}}{(D\ W)^{2n}}} \\
&= \frac{1}{1+\left(\frac{D^2-D_0^2}{D\ W}\right)^{2n}}
\end{aligned}
$$

**bandpass Gaussian filter**

The Gaussian band reject filter is given by

$$
H_{br}(u,v) = 1-\exp\left(-\frac{1}{2}\left(\frac{D^2-D_0^2}{D\ W}\right)\right)
$$

Hence Gaussian band pass filter is

$$
\begin{aligned}
H_{bp}(u,v) &= 1-H_{br}(u,v) \\
&= 1-\left[1-\exp\left(-\frac{1}{2}\left(\frac{D^2-D_0^2}{D\ W}\right)\right)\right] = \exp\left(-\frac{1}{2}\left(\frac{D^2-D_0^2}{D\ W}\right)\right)
\end{aligned}
$$