HW 4, EECS 203A
HW4, Problem 4.5
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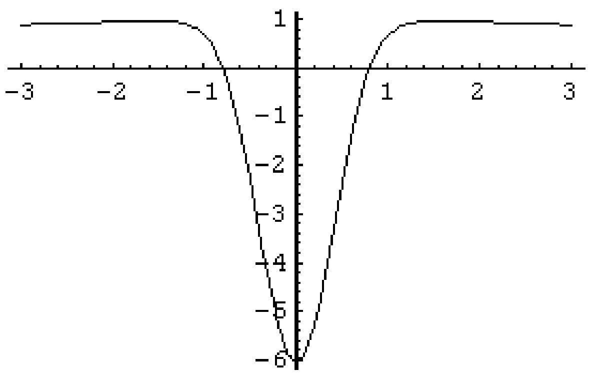
A high pass Gaussian fillter can be consider the difference between 2 Gaussian filters with different σ different amplitude, as shown in this example

In[287]:= $\sigma = 6;\ \alpha = .4;\ A = 7;\ B = 1;$

$$\text{Plot}\left[B\, e^{\frac{-t^2}{2\sigma^2}} - A\, e^{\frac{-t^2}{2\alpha^2}},\ \{t, -3, 3\}\right]$$



ut[288]= - Graphics -

But $H_{gaussian}(u,v) = Ae^{-\frac{(u^2+v^2)}{2\sigma^2}}$

So, let $H_1(u,v) = Ae^{-\frac{(u^2+v^2)}{2\sigma_1^2}}$ and $H_2(u,v) = Be^{-\frac{(u^2+v^2)}{2\sigma_2^2}}$

Hence $H_{hp} = (H_2 - H_1)$
so

$$\begin{aligned} h_{hp}(x,y) &= \int_{v=-\infty}^{\infty}\int_{u=-\infty}^{\infty} (H_2 - H_1)\ \ e^{j2\pi(ux+vy)}\ du\ dv \\ &= \int_{v=-\infty}^{\infty}\int_{u=-\infty}^{\infty} H_2\ e^{j2\pi(ux+vy)} du\ dv - \int_{v=-\infty}^{\infty}\int_{u=-\infty}^{\infty} H_1\ \ e^{j2\pi(ux+vy)}\ du\ dv \end{aligned}$$

But from problem 4.4 result, $\int_{v=-\infty}^{\infty}\int_{u=-\infty}^{\infty} Ae^{-\frac{(u^2+v^2)}{2\sigma^2}}\ \ e^{j2\pi(ux+vy)}\ du\ dv = 2A\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2+y^2)}$

Hence

$$h_{hp}(x,y) = 2B\pi\sigma_2^2 e^{-2\pi^2\sigma_2^2(x^2+y^2)} - 2A\pi\sigma_1^2 e^{-2\pi^2\sigma_1^2(x^2+y^2)}$$

Where $A \geq B$ and $\sigma_1 \geq \sigma_2$