HW2, Problem 3.1
EECS 203A, UCI. Fall 2004
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Part(a).

$$s = e^{-\alpha r^2} \tag{1}$$

When $r = L_0$, then $s = \frac{A}{2}$
Hence

$$\begin{aligned} \frac{A}{2} &= e^{-\alpha L_0^2} \\ \ln\left(\frac{A}{2}\right) &= -\alpha L_0^2 \\ \alpha &= -\frac{\ln\left(\frac{A}{2}\right)}{L_0^2} \end{aligned}$$

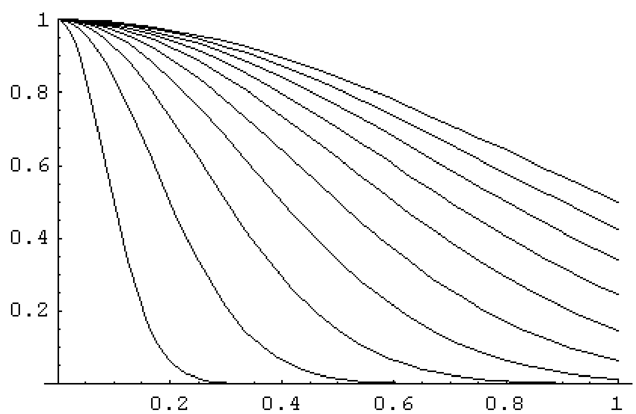
Then (1) becomes

$$\boxed{s = T(r) = e^{\frac{\ln\left(\frac{A}{2}\right)}{L_0^2} r^2}} \tag{2}$$

Below, I wrote an Mathematica function to plot this transformation for different values of L_0, and for $A = .5$ starting from .1 to 1 by increments of .1

```
<< Utilities`Notation`
Clear["Global`*"];
Symbolize[L0];
```

```
n[27]:= s[L0 : _, r_] := e^((Log[1/2]/L0^2) r^2)
data = Table[s[L0, r], {L0, .1, 1, .1}];
Plot[Evaluate[data], {r, 0, 1}, PlotRange → {0, 1}];
```



This function below in maple plots the curves for different values of A and L_0

```
# HW2, Problem 3.1 parta
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restart;
with(plots);
s:= (A,L,r)-> exp( (log(A/2)*r^2)/L^2 );

B:=array(1..3,1..3);

i:=0;
j:=0;

for A from .1 to .8 by .3 do
```
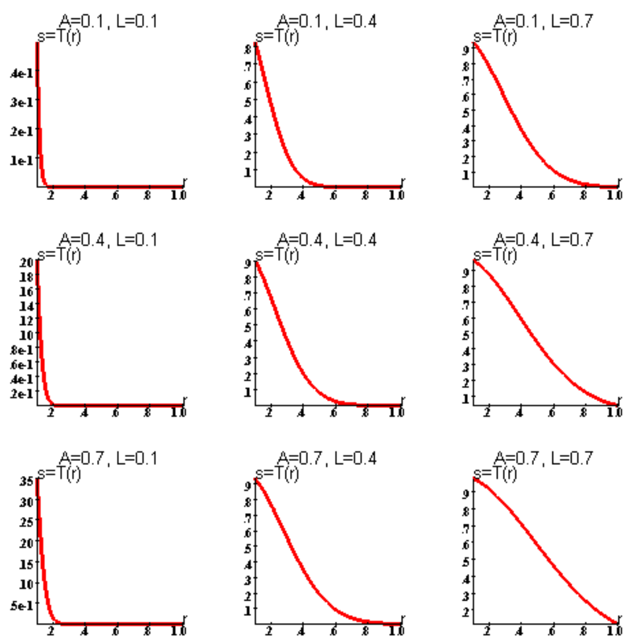

```
    i:=i+1;
    j:=0;
    for L from .1 to .8 by .3 do
        j:=j+1;
        t:=sprintf("A=%1.1f, L=%1.1f",A,L);
        B[i,j]:=plot(s(A,L,r),r=.1..1,labels=["r", "s=T(r)"],title=t,
                     view=[0..1,0..1],axesfont=[TIMES,BOLD,7],
                     scaling=constrained,thickness=3
                     );
    end do;
end do;

display(B,view=[0..1,0..1]);
```



Part(b).

In this case when $r=1$, $s=B$, hence from $s=e^{\frac{\ln\left(\frac{B}{2}\right)}{L_0^2}r^2}$, we get

$$
\begin{aligned}
B &= e^{\frac{\ln\left(\frac{B}{2}\right)}{L_0^2}1^2} \\
\ln B &= \frac{\ln\left(\frac{B}{2}\right)}{L_0^2} = \frac{\ln(B)-\ln(2)}{L_0^2} \\
L_0^2 \ln B - \ln B &= -\ln(2) \\
\ln B\ (L_0^2 - 1) &= -\ln 2 \\
\ln B &= \frac{-\ln 2}{(L_0^2-1)} \\
B &= e^{\frac{-\ln 2}{(L_0^2-1)}}
\end{aligned}
$$

So the transformation is

$$
\boxed{s = T(r) = e^{\frac{\ln\left(\frac{B}{2}\right)}{L_0^2}r^2}}
$$

Where $B = e^{\frac{-\ln 2}{(L_0^2-1)}}$

Below is a maple procedure which displays this transformation

```
# HW2, Problem 3.1 part b
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restart;
with(plots);
s:= (B,L,r)-> exp( (log(B/2)*r^2)/L^2 );

garray:=array(1..3,1..3);

i:=1;
j:=0;

   for L from .1 to .9 by .1 do
       j:=j+1;
```
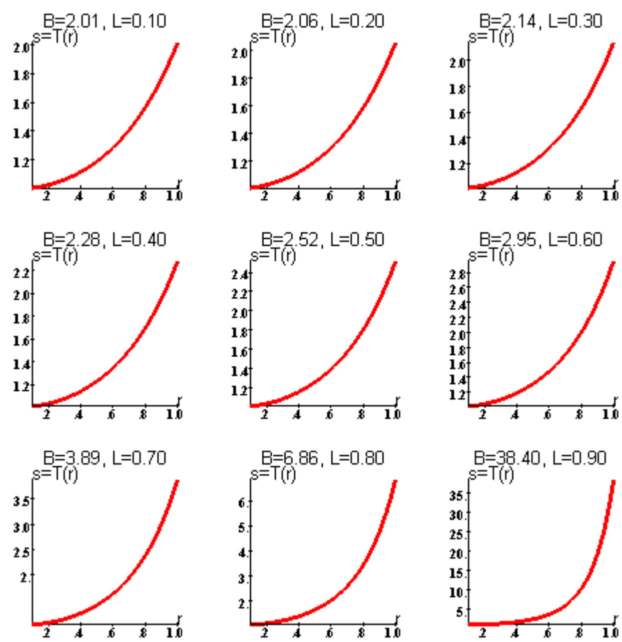

```
        if j>3 then
           j:=1;
           i:=i+1;
        end if;
        i; j;
        B:=exp(-log(2)/(L^2-1));
        t:=sprintf("B=%3.2f, L=%3.2f",B,L);
        garray[i,j]:=plot(s(B,L,r),r=.1..1,labels=["r", "s=T(r)"],title=t,
                     view=[0..1,0..1],axesfont=[TIMES,BOLD,7],
                     scaling=constrained,thickness=3
                     ):
    end do:

display(garray);
```



Part(c)

$$s = C + e^{-\alpha r^2}$$

When $r = 1$, $s = D$, hence

$$\begin{aligned} D &= C + e^{-\alpha} \\ \ln(D - C) &= -\alpha \end{aligned}$$

hence the transformation is

$$\boxed{s = C + e^{\ln(D-C) r^2}}$$