96/100

HW#5

EECS 152A

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HW ~~4~~ 5, EECS 152A DSP.
Problem 4.27 Nasser Abbasi
UCI, Fall 2004.

Question

Determine and sketch the magnitude and phase diagrams for the following systems

(a)$y(n) = \frac{1}{2}[x(n) + x(n-1)]$
(b)$y(n) = \frac{1}{2}[x(n) - x(n-1)]$
(c)$y(n) = \frac{1}{2}[x(n+1) - x(n-1)]$
(d)$y(n) = \frac{1}{2}[x(n+1) + x(n-1)]$
(e)$y(n) = \frac{1}{2}[x(n) + x(n-2)]$
(f)$y(n) = \frac{1}{2}[x(n) - x(n-2)]$
(g)$y(n) = \frac{1}{3}[x(n) + x(n-1) + x(n-2)]$
(h)$y(n) = x(n) - x(n-8)$

Solution

part(a)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n) + \delta(n-1)]$

So, we get values only for $n = 0, 1$ i.e. $h = \left\{\boxed{\frac{1}{2}}, \frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$X(\omega) = \sum\limits_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ we get $H(\omega) = \sum\limits_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum\limits_{n=0}^{1} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2} + \frac{1}{2}e^{-j\omega}}$

i.e. $\boxed{H(\omega) = \frac{1}{2} + \frac{1}{2}e^{-j\omega} = \frac{1}{2} + \frac{1}{2}(\cos\omega - j\sin\omega) = \left(\frac{1}{2} + \frac{1}{2}\cos\omega\right) + j\left(-\frac{1}{2}\sin\omega\right)}$

$|H(\omega)| = \sqrt{\left(\frac{1}{2} + \frac{1}{2}\cos\omega\right)^2 + \left(-\frac{1}{2}\sin\omega\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}\cos^2\omega + \frac{1}{4}\cos^2\omega + \frac{1}{4}\sin^2\omega}$

$= \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}\cos\omega} = \boxed{\sqrt{\frac{1}{2}(1 + \cos\omega)}}$

So at $\omega = 0$, $|H(0)| = \sqrt{\frac{1}{2}(1 + \cos 0)} = 1$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2}(1 + \cos\pi)} = 0$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2}\left(1 + \cos\frac{\pi}{2}\right)} = \sqrt{\frac{1}{2}} = 0.707\,11$

For the phase, we have

$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\frac{1}{2}\sin\omega}{\frac{1}{2}+\frac{1}{2}\cos\omega}\right) = \boxed{\tan^{-1}\left(\frac{-\sin\omega}{1+\cos\omega}\right)}$

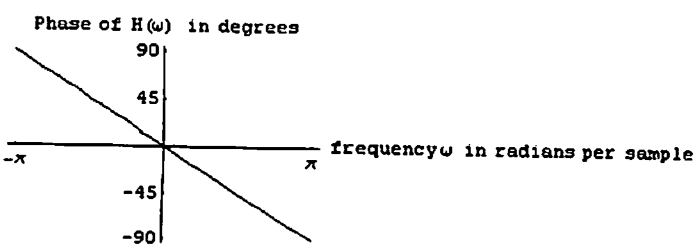
Phase diagram is an odd function and symmetrical across the y-axis. Look at few values, then I show the plot:

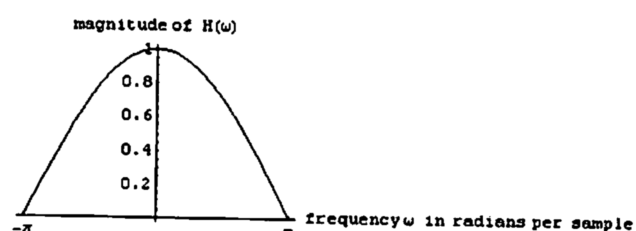
When $\omega = 0$,$\Theta(0) = \tan^{-1}\left(\frac{0}{2}\right) = 0^0$

at $\omega = \pi, \Theta(\pi) = \tan^{-1}\left(\frac{-\sin\pi}{1+\cos\pi}\right) = \tan^{-1}\left(\frac{0}{0}\right) =$ undefined, so point of discontinuity

at $\omega = \frac{\pi}{2}, \Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\left(\frac{-\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = \frac{\pi}{4} = 45^0$

A plot of phase and $|H(\omega)|$ is below





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part(b)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n=0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n) - \delta(n-1)]$

So, we get values only for $n = 0, 1$ i.e. $h = \left\{\boxed{\frac{1}{2}}, -\frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \quad \text{we get } H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=0}^{1} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2} - \frac{1}{2}e^{-j\omega}}$$

i.e. $H(\omega) = \frac{1}{2} - \frac{1}{2}e^{-j\omega} = \frac{1}{2} - \frac{1}{2}(\cos\omega - j\sin\omega)$

$$= \boxed{\left(\tfrac{1}{2} - \tfrac{1}{2}\cos\omega\right) + j\left(\tfrac{1}{2}\sin\omega\right)}$$

$$|H(\omega)| = \sqrt{\left(\tfrac{1}{2} - \tfrac{1}{2}\cos\omega\right)^2 + \left(\tfrac{1}{2}\sin\omega\right)^2} = \sqrt{\tfrac{1}{4} + \tfrac{1}{4}\cos^2\omega - \tfrac{1}{2}\cos\omega + \tfrac{1}{4}\sin^2\omega}$$

$$= \sqrt{\tfrac{1}{2} - \tfrac{1}{2}\cos\omega} = \boxed{\sqrt{\tfrac{1}{2}(1 - \cos\omega)}}$$

So at $\omega = 0$, $|H(\omega)| = 0$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2}(1 - \cos\pi)} = 1$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2}\left(1 - \cos\frac{\pi}{2}\right)} = \sqrt{\frac{1}{2}} = 0.70711$

A plot of $|H(\omega)|$ is shown below

For the phase, we have

$$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{\frac{1}{2}\sin\omega}{\frac{1}{2} - \frac{1}{2}\cos\omega}\right) = \boxed{\tan^{-1}\left(\frac{\sin\omega}{1-\cos\omega}\right)}$$

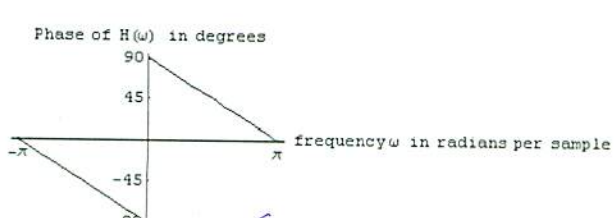
Phase diagram is an odd function and symmetrical across the y-axis. Look at few values, then I show the plot:

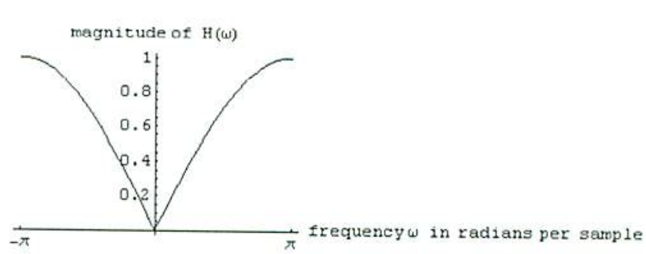
When $\omega = 0$, $\Theta(0) = \tan^{-1}\left(\frac{0}{0}\right)$ undefined, so discontinuity point

at $\omega = \pi$, $\Theta(\pi) = \tan^{-1}\left(\frac{\sin\pi}{1-\cos\pi}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$

at $\omega = \frac{\pi}{2}$, $\Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\left(\frac{\sin\frac{\pi}{2}}{1-\cos\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4} = 45^0$

A plot of the phase is shown below





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Part(c)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n+1) - \delta(n-1)]$

So, we get values only for $n = -1, 1$ i.e. $h = \left\{\frac{1}{2}, \boxed{0}, -\frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=-1}^{1} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2}e^{j\omega} - \frac{1}{2}e^{-j\omega}} = \boxed{j\sin(\omega)}$

$\boxed{|H(\omega)| = |\sin(\omega)|}$ This is just the magnitude of the sin function, which we know how it looks.

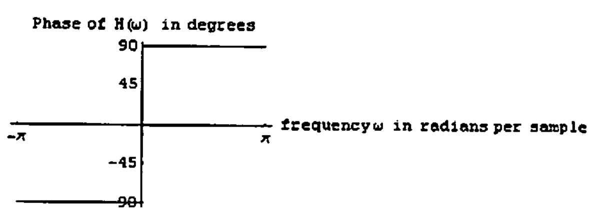
A plot of $|H(\omega)|$ is shown below

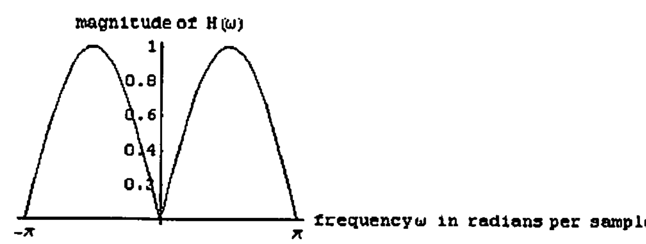
For the phase, since the complex number has only an imaginary part, its phase can only be $\pm 90^0$

When $0 < \omega < \pi$, $\sin(\omega)$ is positive, so $H(\omega)$ on the positive imaginary axis, i.e. phase is $+90^0$

When $-\pi < \omega < 0$, $\sin(\omega)$ is negative, so $H(\omega)$ on the negative imaginary axis, i.e. phase is -90^0

A plot of the phase is shown below





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Part(d)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n+1) + \delta(n-1)]$

So, we get values only for $n = -1, 1$ i.e. $h = \left\{\frac{1}{2}, \boxed{0}, \frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

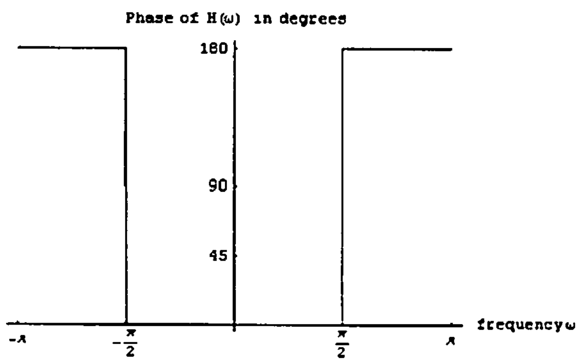
$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=-1}^{1} h(n) e^{-j\omega n} = \boxed{\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}} = \boxed{\cos(\omega)}$

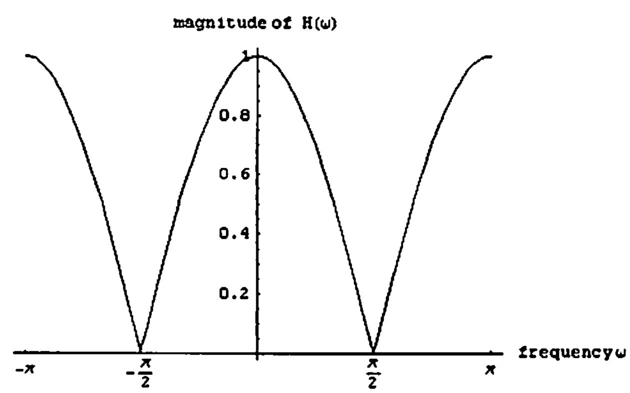
This is just the cos function. $|H(\omega)| = |\cos(\omega)| = 1$ at $\omega = 0, \pm\pi$ and 0 at $\pm\frac{\pi}{2}$

This complex number has only real part, so its phase can be either a zero or 180^0 depending if the real part is positive or negatibe. when $0 < \omega < \frac{\pi}{2}$, $\cos(\omega)$ is positive, so $H(\omega)$ phase is zero. When $\frac{\pi}{2} < \omega < \pi$, then $\cos(\omega)$ is negative, so $H(\omega)$ phase is 180^0

when $-\frac{\pi}{2} < \omega < 0$, $\cos(\omega)$ is positive so phase is zero, when $-\pi < \omega < -\frac{\pi}{2}$, $\cos(\omega)$ is negative so phase is 180

A plot of the phase and magnitude is shown below





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Part(e)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n) + \delta(n-2)]$

So, we get values only for $n = 0, 2$ i.e. $h = \left\{\boxed{\frac{1}{2}}, 0, \frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \text{ we get } H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=0}^{2} h(n)\, e^{-j\omega n} = \boxed{\tfrac{1}{2} + \tfrac{1}{2}e^{-2j\omega}}$$

$$= \tfrac{1}{2} + \tfrac{1}{2}(\cos 2\omega - j\sin 2\omega) = \boxed{\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\omega\right) + j\left(-\tfrac{1}{2}\sin 2\omega\right)}$$

$$|H(\omega)| = \sqrt{\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\omega\right)^2 + \left(-\tfrac{1}{2}\sin 2\omega\right)^2} = \sqrt{\left(\tfrac{1}{4} + \tfrac{1}{4}\cos^2 2\omega + \tfrac{1}{2}\cos 2\omega\right) + \tfrac{1}{4}\sin^2 2\omega}$$

$$= \boxed{\sqrt{\tfrac{1}{2} + \tfrac{1}{2}\cos 2\omega}}$$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = 0$, $|H(0)| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 0} = 1$

at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\pi} = 1$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\frac{\pi}{2}} = 0$

at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{\frac{1}{2} + \frac{1}{2}\cos 2\frac{\pi}{4}} = \sqrt{\frac{1}{2}} = 0.70711$

A plot of $|H(\omega)|$ is shown below

For the phase, we have

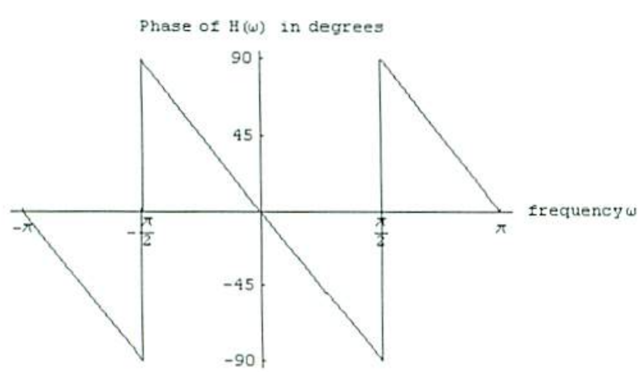
$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\frac{1}{2}\sin 2\omega}{\frac{1}{2}+\frac{1}{2}\cos 2\omega}\right) = \boxed{\tan^{-1}\left(\frac{-\sin 2\omega}{1+\cos 2\omega}\right)}$

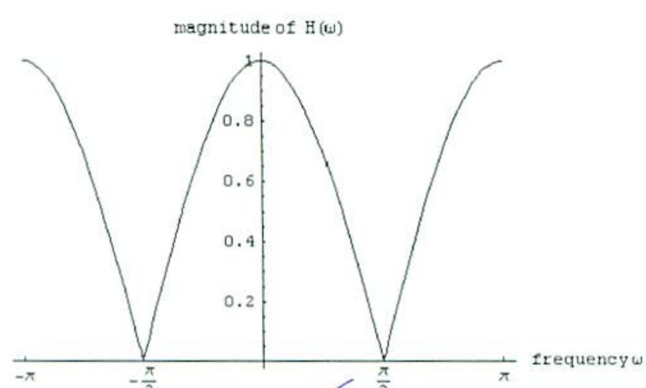
at $\omega = 0$, $\Theta(\omega) = \tan^{-1}\left(\frac{-\sin 0}{1+\cos 0}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$

at $\omega = \pi$, $\Theta(\omega) = \tan^{-1}\left(\frac{-\sin 2\pi}{1+\cos 2\pi}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$

at $\omega = \frac{\pi}{2}$, $\Theta(\omega) = \tan^{-1}\left(\frac{-\sin \pi}{1+\cos \pi}\right) = \tan^{-1}\left(\frac{0}{0}\right) =$ undefined, discontinuity

A plot of the magnitude and phase are below





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Part(f)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \frac{1}{2}[\delta(n) - \delta(n-2)]$

So, we get values only for $n = 0, 2$ i.e. $h = \left\{\boxed{\frac{1}{2}}, 0, -\frac{1}{2}\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \text{ we get } H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n} = \sum_{n=0}^{2} h(n)\, e^{-j\omega n} = \boxed{\frac{1}{2} - \frac{1}{2}e^{-2j\omega}}$$

so $H(\omega) = \frac{1}{2} - \frac{1}{2}(\cos 2\omega - j\sin 2\omega) = \boxed{\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega\right) + j\left(\frac{1}{2}\sin 2\omega\right)}$

$$|H(\omega)| = \sqrt{\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\omega\right)^2 + \left(\tfrac{1}{2}\sin 2\omega\right)^2} = \sqrt{\left(\tfrac{1}{4} + \tfrac{1}{4}\cos^2 2\omega - \tfrac{1}{2}\cos 2\omega\right) + \left(\tfrac{1}{4}\sin^2 2\omega\right)}$$

$$= \boxed{\sqrt{\tfrac{1}{2} - \tfrac{1}{2}\cos 2\omega}}$$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = 0$, $|H(0)| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 0} = 0$

at $\omega = \pi$, $|H(\pi)| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\pi} = 0$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\frac{\pi}{2}} = 1$

at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\frac{\pi}{4}} = \sqrt{\frac{1}{2}} = 0.70711$

A plot of $|H(\omega)|$ is shown below

For the phase, we have

$\Theta(\omega) = \tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))} = \tan^{-1}\left(\frac{\frac{1}{2}\sin 2\omega}{\frac{1}{2} - \frac{1}{2}\cos 2\omega}\right) = \boxed{\tan^{-1}\left(\frac{\sin 2\omega}{1-\cos 2\omega}\right)}$

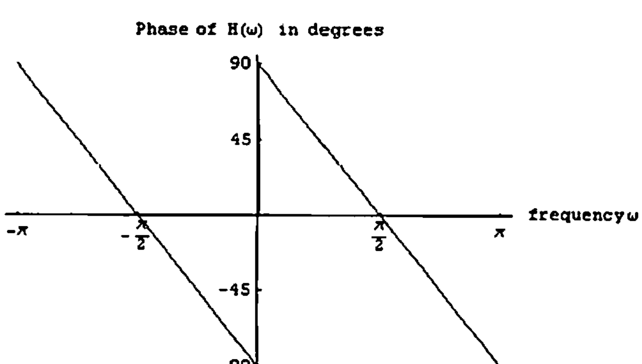
at $\omega = 0$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin 0}{1-\cos 0}\right) = \tan^{-1}\left(\frac{0}{0}\right)$ =undefined, discontinuity point

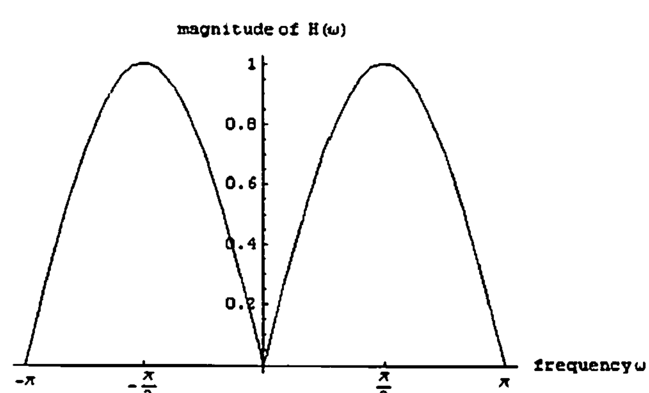
at $\omega = \pi$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin 2\pi}{1-\cos 2\pi}\right) = \tan^{-1}\left(\frac{0}{0}\right)$ =undefined, discontinuity point

at $\omega = \frac{\pi}{2}$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin \pi}{1-\cos \pi}\right) = \tan^{-1}\left(\frac{0}{2}\right) = 0$

at $\omega = \frac{\pi}{4}$, $\Theta(\omega) = \tan^{-1}\left(\frac{\sin \frac{\pi}{2}}{1-\cos\frac{\pi}{2}}\right) = \tan^{-1}\left(\frac{1}{1}\right) = 45^0$

A plot of the magnitude and phase are below





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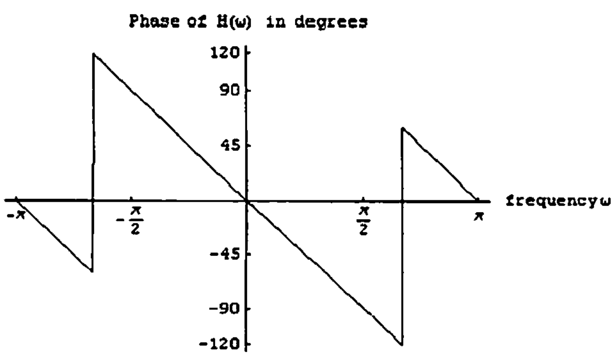
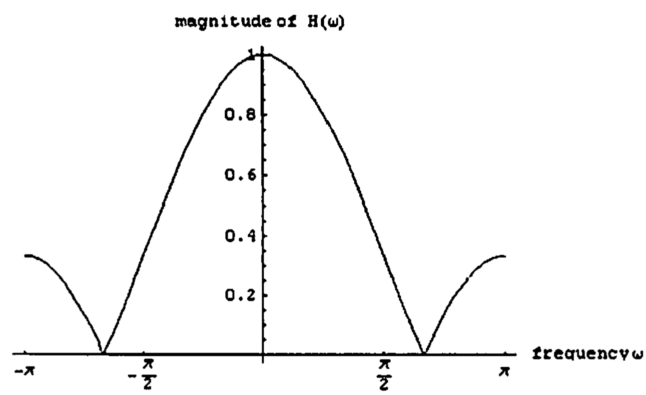
Part(g)
$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n=0$
Hence, replace $x(n)$ with $\delta(n)$ in the above, we get
$h(n)=\frac{1}{3}[\delta(n)+\delta(n-1)+\delta(n-2)]$
So, we get values only for $n=0,1,2$ i.e. $h=\left\{\boxed{\frac{1}{3}},\frac{1}{3},\frac{1}{3}\right\}$
using the Fourier transform for a discrete aperiodic sequence given by
$X(\omega)=\sum\limits_{n=-\infty}^{\infty}x(n)\,e^{-j\omega n}$ we get $H(\omega)=\sum\limits_{n=-\infty}^{\infty}h(n)\,e^{-j\omega n}=\sum\limits_{n=0}^{2}h(n)\,e^{-j\omega n}=\boxed{\frac{1}{3}+\frac{1}{3}e^{-j\omega}+\frac{1}{3}e^{-2j\omega}}$
so $H(\omega)=\frac{1}{3}+\frac{1}{3}(\cos\omega-j\sin\omega)+\frac{1}{3}(\cos 2\omega-j\sin 2\omega)=\frac{1}{3}+\frac{1}{3}\cos\omega-\frac{1}{3}j\sin\omega+\frac{1}{3}\cos 2\omega-\frac{1}{3}j\sin 2\omega$
$=\boxed{\left(\frac{1}{3}+\frac{1}{3}\cos\omega+\frac{1}{3}\cos 2\omega\right)+j\left(-\frac{1}{3}\sin\omega-\frac{1}{3}\sin 2\omega\right)}$
$|H(\omega)|=\sqrt{\left(\frac{1}{3}+\frac{1}{3}\cos\omega+\frac{1}{3}\cos 2\omega\right)^2+\left(-\frac{1}{3}\sin\omega-\frac{1}{3}\sin 2\omega\right)^2}=\boxed{\frac{1}{3}\sqrt{(1+2\cos\omega)^2}}$
I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.
at $\omega=0$, $|H(0)|=\frac{1}{3}\sqrt{(1+2\cos 0)^2}=1$
at $\omega=\pi$, $|H(\pi)|=\frac{1}{3}\sqrt{(1+2\cos\pi)^2}=\frac{1}{3}$
at $\omega=\frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right|=\frac{1}{3}\sqrt{\left(1+2\cos\frac{\pi}{2}\right)^2}=\frac{1}{3}$
at $\omega=\frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right|=\frac{1}{3}\sqrt{\left(1+2\cos\frac{\pi}{4}\right)^2}=\frac{1}{3}\sqrt{\left(1+2\cos\frac{\pi}{4}\right)^2}=0.804738$
A complete plot of $|H(\omega)|$ is shown below

For the phase, we have
$\Theta(\omega)=\tan^{-1}\frac{\mathrm{Im}(H(\omega))}{\mathrm{Re}(H(\omega))}=\tan^{-1}\left(\frac{-\frac{1}{3}\sin\omega-\frac{1}{3}\sin 2\omega}{\frac{1}{3}+\frac{1}{3}\cos\omega+\frac{1}{3}\cos 2\omega}\right)=\boxed{\tan^{-1}\left(\frac{-\sin\omega-\sin 2\omega}{1+\cos\omega+\cos 2\omega}\right)}$
at $\omega=0$, $\Theta(0)=\tan^{-1}\left(\frac{-\sin 0-\sin 0}{1+\cos 0+\cos 0}\right)=\tan^{-1}\left(\frac{0}{3}\right)=0$
at $\omega=\pi$, $\Theta(\pi)=\tan^{-1}\left(\frac{-\sin\pi-\sin 2\pi}{1+\cos\pi+\cos 2\pi}\right)=\tan^{-1}\left(\frac{0}{1}\right)=0$
at $\omega=\frac{\pi}{2}$, $\Theta\left(\frac{\pi}{2}\right)=\tan^{-1}\left(\frac{-\sin\frac{\pi}{2}-\sin 2\frac{\pi}{2}}{1+\cos\frac{\pi}{2}+\cos 2\frac{\pi}{2}}\right)=\tan^{-1}\left(\frac{-1}{1-1}\right)=-90^0$
A plot of the magnitude and phase are below





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Part(h)

$h(n)$ is defined as the output of the system when the input is an impulse $\delta(n)$ which is zero at all n other than $n = 0$

Hence, replace $x(n)$ with $\delta(n)$ in the above, we get

$h(n) = \delta(n) - \delta(n-8)$

So, we get values only for $n = 0, 8$ i.e. $h = \left\{\boxed{1}, 0, 0, 0, 0, 0, 0, 0, 1\right\}$

using the Fourier transform for a discrete aperiodic sequence given by

$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ we get $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{8} h(n) e^{-j\omega n} = \boxed{1 + e^{-8j\omega}} =$

$1 + (\cos 8\omega - j\sin 8\omega) = \boxed{(1 + \cos 8\omega) + j(-\sin 8\omega)}$

$|H(\omega)| = \sqrt{(1+\cos 8\omega)^2 + (\sin 8\omega)^2} = \sqrt{(1 + \cos^2 8\omega + 2\cos 8\omega) + (\sin^2 8\omega)} = \boxed{\sqrt{2 + 2\cos 8\omega}}$

I need to only look at few values from $0..\pi$, since $H(\omega)$ is symmetrical even function w.r.t to the y-axis.

at $\omega = 0$, $|H(0)| = \sqrt{2 + 2\cos 0} = 2$

at $\omega = \pi$, $|H(\pi)| = \sqrt{2 + 2\cos 8\pi} = 2$

at $\omega = \frac{\pi}{2}$, $\left|H\left(\frac{\pi}{2}\right)\right| = \sqrt{2 + 2\cos 8\frac{\pi}{2}} = 2$

at $\omega = \frac{\pi}{4}$, $\left|H\left(\frac{\pi}{4}\right)\right| = \sqrt{2 + 2\cos 8\frac{\pi}{4}} = 2$

at $\omega = \frac{\pi}{3}$, $\left|H\left(\frac{\pi}{3}\right)\right| = \sqrt{2 + 2\cos 8\frac{\pi}{3}} = 1$

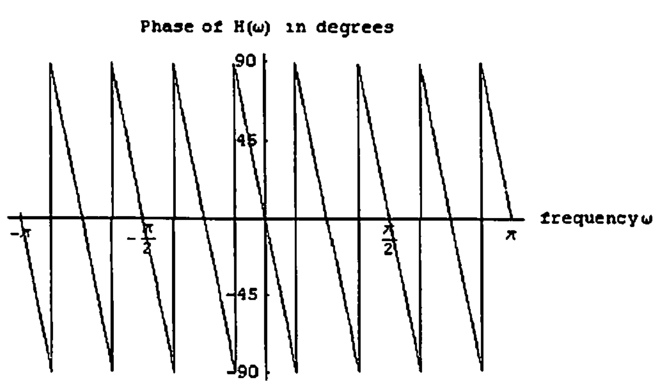
A plot of $|H(\omega)|$ is shown below

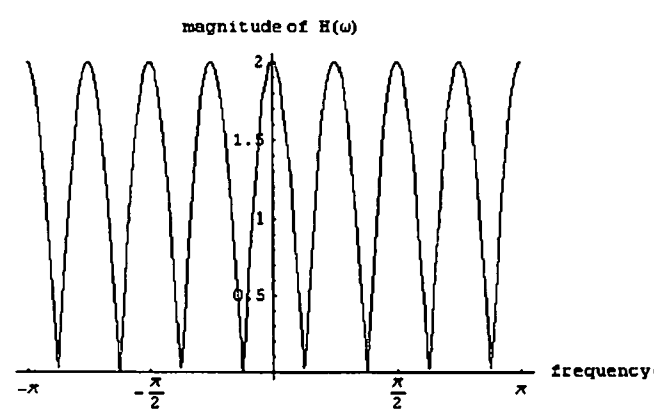
For the phase, we have

$\Theta(\omega) = \tan^{-1}\frac{\text{Im}(H(\omega))}{\text{Re}(H(\omega))} = \tan^{-1}\left(\frac{-\sin 8\omega}{1+\cos 8\omega}\right)$

at $\omega = 0$, $\Theta(0) = \tan^{-1}\left(\frac{0}{2}\right) = 0$

A plot of the magnitude and phase are below





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HW 5, EECS 152A DSP.
Problem 4.28 Nasser Abbasi
UCI, Fall 2004.

Question

An FIR system described by the difference equation $y(n) = x(n) + x(n-10)$

(a)Computer and sketch its magnitude and phase response

(b)Determine its response to the inputs

(1)$x(n) = \cos\left(\frac{\pi n}{10}\right) + 3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right) \quad -\infty < n < \infty$

(2)$x(n) = 10 + 5\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right) \quad -\infty < n < \infty$

Solution

part(a) Take the Z transform of both sides, we get $Y(z) = X(z) + z^{-10}X(Z)$

So, $Y(Z) = X(Z)\left(1 + z^{-10}\right)$ hence $H(Z) = \frac{Y(Z)}{X(Z)} = \left(1 + z^{-10}\right)$

This has a pole at z=0 or order 10, since pole inside unit circle , then stable. Also the Fourier transform exist since ROC defined on the unit circle. To find the Fourier transform, let $z = e^{j\omega}$ hence $1 + z^{-10} = 1 + \left(e^{jw}\right)^{-10}$

Hence $H(\omega) = \boxed{1 + e^{-10j\omega}} = 1 + \cos 10\,\omega - j\sin 10\omega = \boxed{(1 + \cos 10\omega) + j(\sin 10\omega)}$

$|H(\omega)| = \sqrt{(1 + \cos 10\omega)^2 + \sin^2 10\omega} = \sqrt{1 + \cos^2 10\omega + 2\cos 10\omega + \sin^2 10\omega}$

$= \sqrt{2 + 2\cos 10\omega} = \boxed{\sqrt{2(1 + \cos 10\omega)}}$

Try few values: For $\omega = 0, |H(\omega)| = \sqrt{4} = 2$

For $\omega = \frac{\pi}{2}, |H(\omega)| = \sqrt{2(1 + \cos 5\pi)} = \sqrt{2(1-1)} = 0$

A plots for all ω values from -π..π is below.

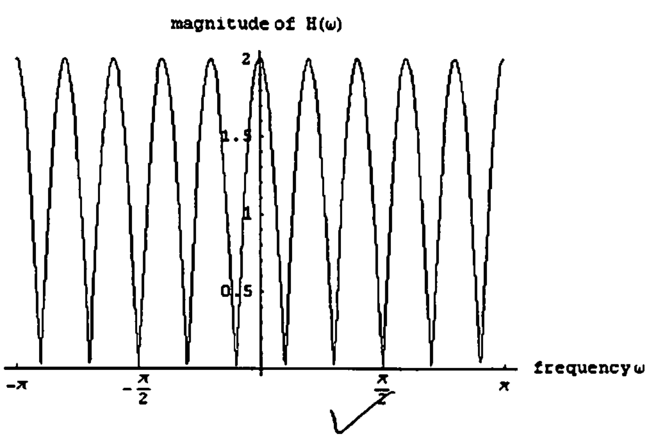
The phase, is given by $\Theta(\omega) = \boxed{\tan^{-1}\frac{\sin 10\omega}{1 + \cos 10\omega}}$

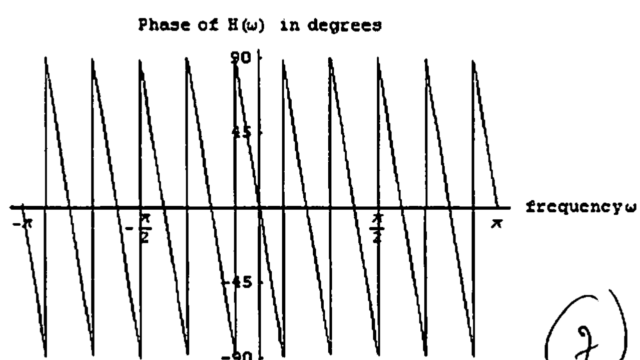
try few values: $\omega = 0, \Theta(\omega) = \tan^{-1}\frac{0}{2} = 0$

$\omega = \pi, \Theta(\pi) = \tan^{-1}\frac{\sin\pi}{1 + \cos 10\pi} = 0$

$\omega = \frac{\pi}{2}, \Theta\left(\frac{\pi}{2}\right) = \tan^{-1}\frac{\sin 10\frac{\pi}{2}}{1 + \cos 10\frac{\pi}{2}}$ =undefined, discontinuity point

A plot for more points is shown below





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part(b)

To find response to $x(n) = \cos\left(\frac{\pi n}{10}\right) + 3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right)$, we note that the input is a combination of complex exponential, hence the output will not modify the frequencies of the input, but will scale the input, and shift the phase. i.e. the input is an eigenfunctions

i.e. if the input is $Ae^{j\omega_1 n}$ then the output is $\boxed{A\ |H(\omega)|\ e^{j(\omega_1 n + \Theta(\omega))}}$ evaluated at $\omega = \omega_1$

From part(a), we have $H(\omega) = 1 + e^{-10j\omega}, |H(\omega)| = \sqrt{2(1+\cos 10\omega)}, \Theta(\omega) = \tan^{-1}\frac{\sin 10\omega}{1+\cos 10\omega}$
First find the input frequencies and phase.
For $\cos\left(\frac{\pi n}{10}\right), \Rightarrow \omega_1 = \frac{\pi}{10}$
so response to this input is

$$\begin{aligned}
y_1(n) &= \sqrt{2(1+\cos 10\omega_1)} \quad \cos\left(\frac{\pi n}{10} + \tan^{-1}\frac{\sin 10\omega_1}{1+\cos 10\omega_1}\right) \\
&= \sqrt{2\left(1+\cos 10\frac{\pi}{10}\right)} \quad \cos\left(\frac{\pi n}{10} + \tan^{-1}\frac{\sin 10\frac{\pi}{10}}{1+\cos 10\frac{\pi}{10}}\right) \\
&= \sqrt{2(1-1)} \ \cos\left(\frac{\pi n}{10} + \tan^{-1}\frac{\sin 10\frac{\pi}{10}}{1+\cos 10\frac{\pi}{10}}\right) \\
&= 0
\end{aligned}$$

So response for $\boxed{\cos\left(\frac{\pi n}{10}\right) \text{ is zero.}}$
Now find the response for $3\sin\left(\frac{\pi n}{3} + \frac{\pi}{10}\right)$, here $\omega_2 = \frac{\pi}{3}$

$$\begin{aligned}
y_2(n) &= 3\sqrt{2(1+\cos 10\omega_2)} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\frac{\sin 10\omega_2}{1+\cos 10\omega_2}\right) \\
&= 3\sqrt{2\left(1+\cos 10\frac{\pi}{3}\right)} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\frac{\sin 10\frac{\pi}{3}}{1+\cos 10\frac{\pi}{3}}\right) \\
&= 3\sqrt{2(1+\cos(240^0))} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\frac{\sin(240^0)}{1+\cos(240^0)}\right) \\
&= 3\sqrt{2\left(1-\frac{1}{2}\right)} \quad \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{1-\frac{1}{2}}\right)\right) \\
&= 3\ \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} + \tan^{-1}\left(-\sqrt{3}\right)\right) \\
&= 3\ \sin\left(\frac{\pi n}{3} + \frac{\pi}{10} - \frac{\pi}{3}\right) \\
&= 3\ \sin\left(\frac{\pi n}{3} + \left(\frac{3-10}{30}\pi\right)\right) \\
&= 3\ \sin\left(\frac{\pi n}{3} - \frac{7}{30}\pi\right)
\end{aligned}$$

$(3) \Rightarrow 6\cos\left(\frac{5\pi}{3}\right) \cdot \sin\left(\frac{\pi n}{3} - \frac{47\pi}{30}\right)$.

Hence the response of the system $\boxed{y(n) = 3\ \sin\left(\frac{\pi n}{3} - \frac{7}{30}\pi\right)}$
(2) $x(n) = 10 + 5\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right)$
The response of the system to the input 10 is simply $|H(\omega)| \times 10$
but $|H(\omega)|$ at $\omega = 0$ is 2, then $y_1(n) = 20$
To find response to $5\cos\left(\frac{2\pi n}{5} + \frac{\pi}{2}\right)$

$$y_2(n) = 5\sqrt{2(1+\cos 10\omega_2)} \quad \cos\left(\frac{2\pi n}{5} + \frac{\pi}{2} + \tan^{-1}\frac{\sin 10\omega_2}{1+\cos 10\omega_2}\right)$$

but $\omega_2 = \frac{2\pi}{5}$

$$
\begin{aligned}
y_2(n) &= 5\sqrt{2\left(1+\cos 10\frac{2\pi}{5}\right)} \quad \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}+\tan^{-1}\frac{\sin 10\frac{2\pi}{5}}{1+\cos 10\frac{2\pi}{5}}\right) \\
&= 10 \quad \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}+\tan^{-1} 0\right) \\
&= 10 \quad \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right)
\end{aligned}
$$

so response of system to $10+5\cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right)$ is $\boxed{20+10\ \cos\left(\frac{2\pi n}{5}+\frac{\pi}{2}\right)}$ ✓ 4

HW5, EECS 152A DSP.
Problem 4.35 Nasser Abbasi
UCI, Fall 2004.

Question
Consider the filter $y(n) = 0.9\ y(n-1) + b\ x(n)$
(a)determine b such that $|H(0)| = 1$
(b)Determine the frequency at which $|H(\omega)| = \frac{1}{\sqrt{2}}$
(c)Is this filter a low pass or high pass
(d)Repeat parts (b),(c) for filter $y(n) = 0.9\ y(n-1) + b\ x(n)$

Solution
part(a) Take the Z transform of both sides, we get

$$\begin{aligned} Y(z) &= 0.9\ z^{-1}Y(z) + bX(z) \\ Y(z) - 0.9\ z^{-1}Y(z) &= bX(Z) \\ Y(z)\left(1 - 0.9\ z^{-1}\right) &= bX(Z) \\ H(z) &= \frac{Y(z)}{X(Z)} \end{aligned}$$

Hence

$$\boxed{H(z) = \frac{b}{(1-0.9\ z^{-1})}}$$

ROC: $0.9z^{-1} < |1|$ or $z > |0.9|$
Hence defined on the unit circle, and Fourier transform exist.
To find the Fourier transform, let $z = e^{j\omega}$ hence

$$\begin{aligned} H(\omega) &= \frac{b}{(1-0.9\ e^{-j\omega})} \\ &= \frac{b}{(1-0.9\ (\cos\omega - j\sin\omega))} \\ &= \frac{b}{(1-0.9\ \cos\omega) + 0.9j\sin\omega}\,\frac{(1-0.9\ \cos\omega) - 0.9j\sin\omega}{(1-0.9\ \cos\omega) - 0.9j\sin\omega} \\ &= \frac{(b-0.9b\ \cos\omega) - 0.9bj\sin\omega}{(1-0.9\ \cos\omega)^2 - (0.9j\sin\omega)^2} \\ &= \frac{(b-0.9b\ \cos\omega) - 0.9bj\sin\omega}{1.81 - 1.8\cos\omega} \end{aligned}$$

✓

So $\mathrm{Re}(H) = \frac{(b-0.9b\ \cos\omega)}{1.81-1.8\cos\omega}$ and $\mathrm{Im}(H) = \frac{-0.9b\sin\omega}{1.81-1.8\cos\omega}$
Hence

$$\begin{aligned} |H(\omega)| &= \sqrt{\left(\frac{(b-0.9b\ \cos\omega)}{1.81-1.8\cos\omega}\right)^2 + \left(\frac{0.9b\sin\omega}{1.81-1.8\cos\omega}\right)^2} \\ &= \sqrt{\frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2}} \end{aligned}$$

Let $\omega = 0$ and solve for b

✓

$$|H(0)| = \sqrt{\frac{(b-0.9b)^2}{(1.81-1.8)^2}} = \sqrt{\frac{0.01b^2}{0.0001}} = \sqrt{\frac{b^2}{0.01}} = 10\,b$$

so

$$10\,b = 1$$

then

$$\boxed{b = \frac{1}{10}}$$

(5)

part(b)

solve for ω

$$\begin{aligned}
\frac{1}{\sqrt{2}} &= |H(\omega)| == \sqrt{\frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2}} \\
\frac{1}{2} &= \frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2} \\
(1.81-1.8\cos\omega)^2 &= 2(b-0.9b\ \cos\omega)^2 + 2(0.9b\sin\omega)^2 \\
3.24\cos^2\omega - 6.516\cos\omega + 3.2761 &= 2b^2 - 3.6b^2\cos\omega + 1.62b^2\cos^2\omega + 1.62b^2\sin^2\omega \\
0 &= 2b^2 - 3.6b^2\cos\omega + 3.24b^2 \ - 3.24\cos^2\omega + 6.516\cos\omega - 3.2761
\end{aligned}$$

Let $\cos\omega = x$

$$0 = 2b^2 - 3.6b^2x + 3.24b^2 \ - 3.24x^2 + 6.516x - 3.2761$$

with help of computer, Solution is:
$1.0056 - 0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2$,
$0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2 + 1.0056$
i.e.

$\boxed{\omega = \arccos\left(1.0056 - 0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2\right)}$

$\omega = \arccos\left(0.15432\sqrt{20.995b^2 + 12.96b^4} - 0.55556b^2 + 1.0056\right)$

for example, at $b = .1$ we get $\omega = \underline{0.105}$

$\omega = \arccos\left(1.0056 - 0.15432\sqrt{20.995(.1)^2 + 12.96(.1)^4} - 0.55556(.1)^2\right) = \underline{0.37878}$

(4)

part(c)

$|H(\omega)| = \sqrt{\frac{(b-0.9b\ \cos\omega)^2 + (0.9b\sin\omega)^2}{(1.81-1.8\cos\omega)^2}}$

at $\omega = \pi$ we have, when $b = (0.1)$, $|H(\pi)| = \sqrt{\frac{((0.1)-0.9(0.1)\ \cos\pi)^2 + (0.9(0.1)\sin\pi)^2}{(1.81-1.8\cos\pi)^2}} = 5.2632\times 10^{-2}$

Hence we see that $|H(\omega)|$ is much smaller at high frequency than at DC, hence this is $\boxed{\text{low pass}}$

(5)

part(d)

Take the Z transform of both sides, we get

$$\begin{aligned}
Y(z) &= -0.9\,z^{-1}Y(z) + 0.1X(z) \\
Y(z) + 0.9\,z^{-1}Y(z) &= 0.1X(Z) \\
Y(z)\left(1 + 0.9\,z^{-1}\right) &= 0.1X(Z) \\
H(z) &= \frac{Y(z)}{X(Z)}
\end{aligned}$$

Hence

$$\boxed{H(z) = \frac{0.1}{(1+0.9\ z^{-1})}}$$

To find the Fourier transform, let $z = e^{j\omega}$ hence

$$\begin{aligned}
H(\omega) &= \frac{.1}{(1+0.9\ e^{-j\omega})} \\
&= \frac{.1}{(1+0.9\ (\cos\omega - j\sin\omega))} \\
&= \frac{.1}{(1+0.9\ \cos\omega) - 0.9j\sin\omega} \frac{(1+0.9\ \cos\omega) + 0.9j\sin\omega}{(1+0.9\ \cos\omega) + 0.9j\sin\omega} \\
&= \frac{(.1 - 0.09\ \ \cos\omega) + 0.09j\sin\omega}{(1+0.9\ \cos\omega)^2 - (0.9j\sin\omega)^2} \\
&= \frac{(.1 - 0.09\ \ \cos\omega) + 0.09j\sin\omega}{1.8\cos\omega + 1.81}
\end{aligned}$$

so

So $\operatorname{Re}(H) = \frac{.1-0.09\ \ \cos\omega}{1.81+1.8\cos\omega}$ and $\operatorname{Im}(H) = \frac{0.09\sin\omega}{1.81+1.8\cos\omega}$

$$\begin{aligned}
|H(\omega)| &= \sqrt{\left(\frac{.1-0.09\ \ \cos\omega}{1.81+1.8\cos\omega}\right)^2 + \left(\frac{0.09\sin\omega}{1.81+1.8\cos\omega}\right)^2} \\
\frac{1}{2} &= \left(\frac{.1-0.09\ \ \cos\omega}{1.81+1.8\cos\omega}\right)^2 + \left(\frac{0.09\sin\omega}{1.81+1.8\cos\omega}\right)^2 \\
\frac{1}{2} &= \frac{(.1-0.09\ \ \cos\omega)^2 + (0.09\sin\omega)^2}{(1.81+1.8\cos\omega)^2} \\
\frac{1}{2} &= \frac{0.0181 - 0.018\ \cos\omega}{6.516\cos\omega + 3.24\cos^2\omega + 3.2761} \\
6.516\cos\omega + 3.24\cos^2\omega + 3.2761 &= 2(0.0181 - 0.018\ \cos\omega) \\
6.516\cos\omega + 3.24\cos^2\omega + 3.2761 &= 0.0362 - 0.036\ \cos\omega \\
0 &= -6.552\cos\omega - 3.24\cos^2\omega - 3.2399
\end{aligned}$$

Let $\cos\omega = x$

$0 = -6.552x - 3.24x^2 - 3.2399$, Solution is: $x = -1.1607, x = -0.86152$

i.e. $\boxed{\omega = \arccos(-0.86152) = 2.6091}$ the other root is not used as imaginary

To find if low or high filter, let $\omega = 0$ then

$|H(0)| = \sqrt{\left(\frac{.1-0.09\ \ \cos 0}{1.81+1.8\cos 0}\right)^2 + \left(\frac{0.09\sin 0}{1.81+1.8\cos 0}\right)^2} = \sqrt{\left(\frac{.1-0.09}{1.81+1.8}\right)^2} = 2.7701\times 10^{-3}$

at $\omega = \pi$, $|H(\pi)| = \sqrt{\left(\frac{.1-0.09\ \ \cos\pi}{1.81+1.8\cos\pi}\right)^2 + \left(\frac{0.09\sin\pi}{1.81+1.8\cos\pi}\right)^2} = \sqrt{\left(\frac{.1-0.09\ \ \cos\pi}{1.81+1.8\cos\pi}\right)^2} = 19.0$

Since $|H(\omega)|$ is much larger at large frequency, than at DC, then this is a $\boxed{\text{high pass filter}}$

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HW#5
Problem 4.51
EECS 152 A. Nasser Abbasi

Solve using geometrical argument.

(a) since we want $|H(\omega)|$ to be zero when $\omega=0$, then

since $|H(\omega)| = \dfrac{|ZZ_k|\cdots}{|PP_k|\cdots}$, then we want

$|ZZ_k|$ to be zero at $\omega=0$.

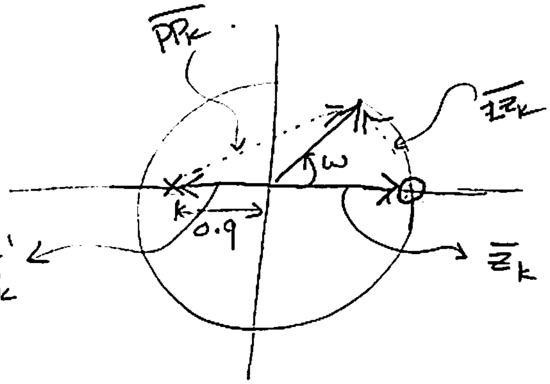
also since we want max response when $\omega=\pm\pi$ (max freq in discrete system), then we want $|PP_k|$ to be smallest at $\omega=\pm\pi$. hence we set

$$H(\omega)=b_0\ \frac{1-Z_k e^{-j\omega}}{1-P_k e^{-j\omega}}$$

$$\boxed{H(\omega)=b_0\ \frac{1}{1-0.9e^{j\pi}e^{-j\omega}}}$$

$$\boxed{H(z)=\frac{b_0}{1+0.9z^{-1}}}$$

there is a zero at P_k



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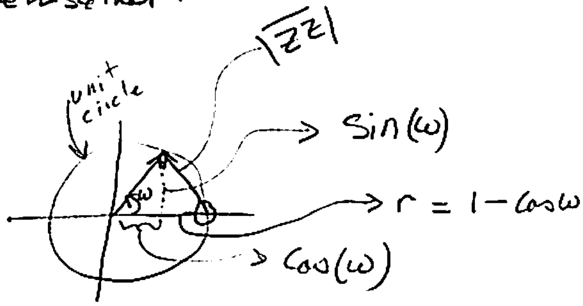
$$\boxed{H(z)=\frac{z-1}{z+0.9}}$$

I call vectors from zero location to tip of $e^{j\omega}$ as $\overline{ZZ_k}$ and vector from pole location to tip of $e^{j\omega}$ as $\overline{PP_k}$. to make it easier to differentia from the actual vector $\overline{Z_k}$ and $\overline{P_k}$ themselves.

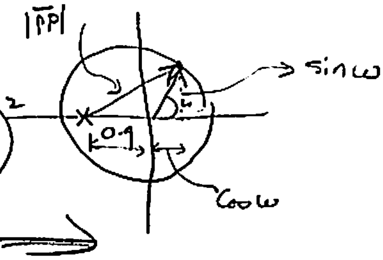
(b) $|H(\omega)| = |b_0| \dfrac{|\overline{ZZ_k}|\cdots}{|\overline{PP_k}|\cdots}$



to find $|\overline{ZZ}|$, use Phythagores theom

$$|\overline{ZZ}|^2 = \sin^2(\omega) + r^2 \quad \text{, but } r = 1-\cos(\omega) \text{ from diagram.}$$

so $|\overline{ZZ}| = \sqrt{(1-\cos(\omega))^2+\sin^2\omega}$



For $|\overline{PP}|$, we see that $|\overline{PP}|^2 = \sin^2\omega + (0.9+\cos\omega)^2$

so $|\overline{PP}| = \sqrt{(0.9+\cos\omega)^2+\sin^2\omega}$ $\Longrightarrow$

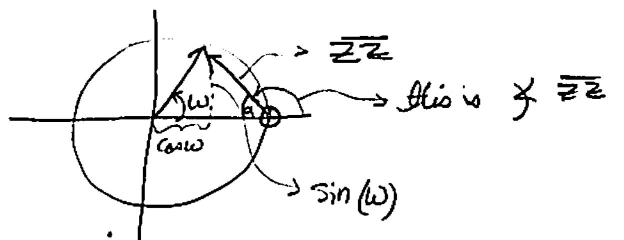
so $$\boxed{|H(\omega)| = |b_0| \frac{\sqrt{(1-\cos\omega)^2+\sin^2\omega}}{\sqrt{(0.9+\cos\omega)^2+\sin^2\omega}}}$$

for phase of $H(\omega)$,

$$\measuredangle H(\omega) = \measuredangle b_0 + \omega(\overset{\#\text{ poles}}{N}-\overset{\#\text{ zeros}\ldots}{M}) + \measuredangle \overline{zz}_k \cdots - (\measuredangle \overline{pp}_k + \cdots)$$

so $\measuredangle H(\omega) = \measuredangle b_0 + \measuredangle \overline{zz} - \measuredangle \overline{pp}$

$\measuredangle \overline{zz}$:

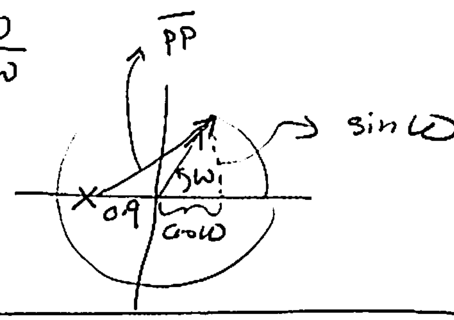


we see that $\measuredangle \overline{zz} = 180^\circ - \alpha$

but $\alpha = \tan^{-1} \frac{\sin\omega}{1-\cos\omega}$.

so $\measuredangle \overline{zz} = \pi - \tan^{-1} \frac{\sin\omega}{1-\cos\omega}$

$\measuredangle \overline{pp}$:



we see that $\measuredangle \overline{pp} = \tan^{-1} \frac{\sin\omega}{0.9+\cos\omega}$

so $$\boxed{\measuredangle H(\omega) = \measuredangle b_0 + \left(\pi - \tan^{-1} \frac{\sin\omega}{1-\cos\omega}\right) - \left(\tan^{-1} \frac{\sin\omega}{0.9+\cos\omega}\right)}$$

(c) need to find $|b_0|$. so that $|H(\omega)| = 1$ when $\omega = \pi$.

$$\left|H(\omega)\right|_{\omega=\pi} = |b_0| \frac{\sqrt{(1-\cos\pi)^2 + \sin^2\pi}}{\sqrt{(0.9+\cos\pi)^2 + \sin^2\pi}} = 1 .$$

so $$|b_0| \frac{\sqrt{(1-(-1))^2}}{\sqrt{(0.9-1)^2}} = 1$$

$$|b_0| \frac{2}{0.1} = 1 \Rightarrow |b_0| = \frac{0.1}{2} = \boxed{0.05}$$

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so $$\boxed{H(\omega) = \frac{0.05}{1+0.9e^{-j\omega}}}$$

(d) since $$H(z) = \frac{0.05}{1+0.9z^{-1}} = 0.05\left(\frac{1}{1+0.9z^{-1}}\right)$$

$$H(z) = 0.05 \frac{1}{1-(-0.9z^{-1})}$$

from table, we see that $a^n u(n) \xrightarrow{Z} \frac{1}{1-az^{-1}}$

so let $a = -0.9$, we get

$$h(n) = 0.05\,(-0.9)^n\,u(n)$$

$$= \boxed{(-1)^n\; 0.05\; (0.9)^n u(n)}$$

$$h = \{\underset{\uparrow}{0.05}, -0.045, 0.0405, -0.03645, \ldots\}$$

did not need to do this actually.

we see from $H(z) = \frac{Y(z)}{X(z)}$ that:

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$$\frac{0.05}{1+0.9z^{-1}} = \frac{Y(z)}{X(z)} \rightarrow 0.05X(z) = Y(z) + 0.9Y(z)z^{-1}$$

$$\Rightarrow \boxed{\begin{aligned} 0.05x(n) &= y(n) + 0.9\;y(n-1) \\ y(n) &= -0.9\,y(n-1) + 0.05\,x(n) \end{aligned}}$$

(e) need to find output if input is

$$x(n) = 2\cos\left(\frac{\pi}{6}n + 45^\circ\right)$$

we see here that $\omega = \frac{\pi}{6}$.

so $y(n) = 2\left|H(\omega=\tfrac{\pi}{6})\right| \cos\left(\frac{\pi}{6}n + 45^\circ + \measuredangle H(\omega=\tfrac{\pi}{6})\right)$

when $\omega = \frac{\pi}{6}$, $|H(\omega)| = 0.05 \dfrac{\sqrt{(1-\cos\frac{\pi}{6})^2 + \sin^2\frac{\pi}{6}}}{\sqrt{(0.9+\cos\frac{\pi}{6})^2 + \sin^2\frac{\pi}{6}}} =$ $\left(\frac{\pi}{6}=30^\circ\right)$

$$= \frac{0.05\,(0.517638)}{1.8354} = \boxed{0.014101}$$

when $\omega = \frac{\pi}{6}$, $\measuredangle H(\omega) = \measuredangle 0.05 + \left(\pi - \tan^{-1}\frac{\sin 30^\circ}{1-\cos 30^\circ}\right) - \left(\tan^{-1}\frac{\sin 30^\circ}{0.9+\cos 30^\circ}\right)$

$= 0$ or π
but $+0.05 \Rightarrow \measuredangle = 0$

so $\measuredangle H(\omega) = 0 + (\pi - 75^\circ) - (15.807^\circ)$

$$= \boxed{189.19^\circ}$$

so $y(n) = 2(0.0141)\cos\left(\frac{\pi}{6}n + 45^\circ + 89.19^\circ\right)$

$$\boxed{y(n) = (0.0282)\cos\left(\frac{\pi}{6}n + 134.19^\circ\right)}$$

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