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HW#4

EECS 152A, Digital Signal processing

UCI. Fall 2004

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HW 4, EECS 152A
Problem 4.5, Nasser Abbasi

Question
consider signal $x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{4} + \frac{1}{2}\cos\frac{3\pi n}{4}$
(a)Determine and sketch its power density spectrum
(b)Evaluate the power of the signal.

Solution
(a) I will use these relations for this problem: $\cos\frac{\pi}{3} = \frac{1}{2}$, $\cos\frac{2\pi}{3} = -\frac{1}{2}$, $\sin\frac{\pi}{3} = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos x = \frac{e^{jx}+e^{-jx}}{2}$

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{4} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

First find the period. For the second term $2\cos\frac{\pi n}{4}$, we get $\frac{\pi n}{4} \equiv 2\pi f n$ hence $f = \frac{1}{8}$ hence periodic (since rational) and period is 8.
For the third term $\cos\frac{\pi n}{4}$, same period.
For the 4th term $\cos\frac{3\pi n}{4}$,we get $\frac{3\pi n}{4} \equiv 2\pi f n$ hence $f = \frac{3}{8}$, hence rational, hence periodic Since lowest common multiplier allready, then period is 8.

$\boxed{\text{Hence the period of } x(n) \text{ is 8.}}$

Hence $x(n)$ can be written as $\boxed{x(n) = 2 + 2\cos\frac{2\pi}{8}n + \cos\frac{2\pi}{8}n + \frac{1}{2}\cos\frac{2\pi}{8}3n}$
Expand in complex exponentials we get

$$\begin{aligned} x(n) &= 2 + 2\left(\frac{e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n}}{2}\right) + \left(\frac{e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n}}{2}\right) + \frac{1}{2}\left(\frac{e^{j\frac{2\pi}{8}3n} + e^{-j\frac{2\pi}{8}3n}}{2}\right) \\ &= 2 + e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}n} + \frac{1}{2}e^{j\frac{2\pi}{8}n} + \frac{1}{2}e^{-j\frac{2\pi}{8}n} + \frac{1}{4}e^{j\frac{2\pi}{8}3n} + \frac{1}{4}e^{-j\frac{2\pi}{8}3n} \end{aligned}$$

Now convert all exponentials to the 'positive' side, so I can compare later with the IDFT. Using the periodicty of complex exponential, we know that

$$e^{-j\frac{2\pi}{8}n} = -e^{j\frac{2\pi}{8}3n}$$
$$e^{-j\frac{2\pi}{8}3n} = -e^{j\frac{2\pi}{8}n}$$

$$e^{-j\frac{2\pi}{8}n} = e^{j\left[-\frac{2\pi}{8}n + 2\pi n\right]} = e^{j2\pi\left(-1/8+1\right)n} = e^{j2\pi(7/8)n}$$

Hence

$$\begin{aligned} x(n) &= 2 + e^{j\frac{2\pi}{8}n} - e^{j\frac{2\pi}{8}3n} + \frac{1}{2}e^{j\frac{2\pi}{8}n} - \frac{1}{2}e^{j\frac{2\pi}{8}3n} + \frac{1}{4}e^{j\frac{2\pi}{8}3n} - \frac{1}{4}e^{j\frac{2\pi}{8}n} \\ &= 2 + \frac{5}{4}e^{j\frac{2\pi}{8}n} - \frac{7}{4}e^{j\frac{2\pi}{8}3n} \end{aligned}$$

Now we know that IDFT is of the form

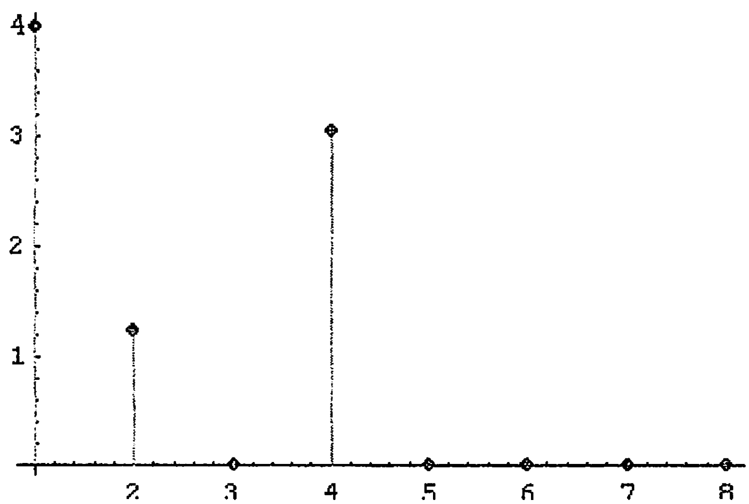
$$x(n) = \sum_{k=0}^{N-1} c(k) e^{j2\pi n\frac{k}{N}}$$

Hence by comparing term by term we see by inspection that
$c(0) = 2$
$c(1) = \frac{5}{4}$
$c(3) = -\frac{7}{4}$
And since $c(k)$ will have the same period as $x(n)$ we then write

$$\boxed{c(k) = \{2, \tfrac{5}{4}, 0, -\tfrac{7}{4}, 0, 0, 0, 0\}}$$

$C_K = \{2, 1, \tfrac{1}{2}, 0, 0, 0, \tfrac{1}{2}, 1\}$

So power density spectrum is
$|c(k)|^2 = \{4, \frac{20}{16}, 0, \frac{49}{16}, 0, 0, 0, 0\}$
This is a sketch of the power spectrum. y-axes is $|c(k)|^2$, and x-axis is k.



(b) Power of signal is given by $\sum_{k=0}^{N-1} |c(k)|^2 = 4 + \frac{20}{16} + 0 + \frac{49}{16} = \boxed{8.3125}$

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Problem 4.7 part(a), Nasser Abbasi

Question
Determine the periodic signal $x(n)$ with fundamental period $N = 8$ if their fourier coefficients are given by
(a) $c(k) = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

Solution
(a) I will use these relations for this problem
$\cos x = \frac{e^{jx}+e^{-jx}}{2}$, $\sin x = \frac{e^{jx}-e^{-jx}}{2j}$

$$c(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j2\pi\frac{n}{N}k} \tag{1}$$

Expand given $c(k)$ in terms of complex exponentials, and compare terms to find $x(n)$

$$\begin{aligned} c(k) &= \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4} \\ &= \cos\frac{2\pi}{8}k + \sin\frac{2\pi}{8}3k \\ &= \frac{e^{j\frac{2\pi}{8}k} + e^{-j\frac{2\pi}{8}k}}{2} + \frac{e^{j\frac{2\pi}{8}3k} - e^{-j\frac{2\pi}{8}3k}}{2j} \\ &= \frac{1}{2}e^{j\frac{2\pi}{8}k} + \frac{1}{2}e^{-j\frac{2\pi}{8}k} + \frac{1}{2j}e^{j\frac{2\pi}{8}3k} - \frac{1}{2j}e^{-j\frac{2\pi}{8}3k} \quad ✓ \end{aligned} \tag{2}$$

Now write all the exponentials in 'negative' terms, so I can compare with (1).
Using periodicty property, $e^{j\frac{2\pi}{8}k} = -e^{-j\frac{6\pi}{8}k} = -e^{-j\frac{2\pi}{8}3k}$
and $e^{j\frac{2\pi}{8}3k} = e^{j\frac{6\pi}{8}k} = -e^{-j\frac{2\pi}{8}k}$
Hence (2) can be rewritten as

$$\begin{aligned} c(k) &= -\frac{1}{2}e^{-j\frac{2\pi}{8}3k} + \frac{1}{2}e^{-j\frac{2\pi}{8}k} - \frac{1}{2j}e^{-j\frac{2\pi}{8}k} - \frac{1}{2j}e^{-j\frac{2\pi}{8}3k} \\ &= \boxed{e^{-j\frac{2\pi}{8}3k}\left(-\frac{1}{2} - \frac{1}{2j}\right) + e^{-j\frac{2\pi}{8}}\left(\frac{1}{2} - \frac{1}{2j}\right)} \quad ✓ \end{aligned}$$

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Hence we see that $x(1) = 8\left(\frac{1}{2} - \frac{1}{2j}\right)$ and $x(3) = 8\left(-\frac{1}{2} - \frac{1}{2j}\right)$
Can also be written as $\boxed{x(1) = (4+4j) \text{ and } x(3) = (-4+4j)}$
or
Hence

$$\boxed{x(n) = \{0,\ (4+4j)\ ,\ 0\ ,\ (-4+4j)\ ,\ 0,\ 0,\ 0,\ 0\}}$$

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Problem 4.9 part(a,b,c), Nasser Abbasi

Question
Compute Fourier transform for the following
(a)$x(n) = u(n) - u(n-6)$
(b)$x(n) = 2^n u(-n)$
(c)$x(n) = \frac{1}{4}^n u(n+4)$
Solution
using the Fourier transform for a discrete aperiodic sequence given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

(a) here $x(n) = \{\hat{1}, 1, 1, 1, 1, 0, 0, \cdots\}$
Hence

$$X(\omega) = \sum_{n=0}^{5} e^{-j\omega n} = \boxed{1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega}}$$

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(b)

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n} = \sum_{0}^{\infty} 2^{-n} e^{j\omega n} = \sum_{0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n = \boxed{\frac{1}{1-\frac{e^{j\omega}}{2}}}$$

(c)

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-4}^{\infty} \frac{1}{4}^n e^{-j\omega n} = \sum_{n=-4}^{-1} \frac{1}{4}^n e^{-j\omega n} + \sum_{0}^{\infty} 2^{-n} e^{j\omega n}$$

$$\begin{aligned} X(\omega) &= \left(\frac{1}{4}^{-4} e^{4\omega j} + \frac{1}{4}^{-3} e^{3\omega j} + \frac{1}{4}^{-2} e^{2\omega j} + \frac{1}{4}^{-1} e^{\omega j} \right) + \sum_{0}^{\infty} \left(\frac{e^{j\omega}}{4}\right)^n \\ &= \boxed{\left(64\, e^{4\omega j} + 32\, e^{3\omega j} + 16\, e^{2\omega j} + 4\, e^{\omega j} \right) + \frac{1}{1-\frac{e^{j\omega}}{4}}} \end{aligned}$$

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