HW#3

EECS 152A, Digital Signal processing

UCI. Fall 2004

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HW#3, EECS 152A.
Problem 1.11

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Statment Consider DSP system

Samplins period of A/D and D/A are $T = 5$ ms, $T' = 1$ ms.
Determine output $y_a(t)$ of system. input is

$$x_a(t) = 3\cos 100\pi t + 2\sin 250\pi t \quad (t \text{ in sec})$$

Postfilter removes any frequency above $F_s/2$.

Solution

$$x(n) = x_a(nT) = 3\cos(100\pi)(nT) + 2\sin 250\pi(nT)$$

but $T = 5$ ms $= 5\times10^{-3}$ sec

$$\text{so } x(n) = 3\cos\left(100\pi\right)\left(\tfrac{5}{1000}n\right) + 2\sin\left(250\pi\right)\left(\tfrac{5}{1000}n\right)$$

$$f_1 \Rightarrow \frac{500\pi n}{1000} \equiv 2\pi f_1 n \Rightarrow f_1 = \frac{500}{2000} = \boxed{\frac{1}{4}} \text{ sample/sec}$$

$$f_2 \Rightarrow \frac{1250\pi n}{1000} \equiv 2\pi f_2 n \Rightarrow f_2 = \frac{1250}{2000} = 0.625 \text{ sample/sec.}$$

$$f_2 > |\tfrac{1}{2}| \Rightarrow \text{alias, so } f_2 = 0.625 - 1 = \boxed{-0.375 \text{ sample/sec}}$$

$$\text{hence } x(n) = 3\cos(2\pi f_1 n) + 2\sin(2\pi f_2 n)$$

$$= 3\cos\left(2\pi \frac{n}{4}\right) + 2\sin\left(2\pi(-0.375)n\right)$$

$$= 3\cos\left(\frac{\pi}{2}n\right) - 2\sin\left(0.75\pi n\right)$$

$$\boxed{x(n) = 3\cos\left(\frac{\pi}{2}n\right) - 2\sin\left(\frac{3}{4}\pi n\right)}$$

✓

or $$\boxed{x(n) = 3\cos\left(2\pi\left(\tfrac{1}{4}\right)n\right) - 2\sin\left(2\pi\left(\tfrac{3}{8}\right)n\right)}$$

For D/A

$T' = 1\text{ ms} \Rightarrow \boxed{F_s' = 1000\ \text{Hz}}$

so $\frac{F_1'}{F_s'} = f_1 \Rightarrow F_1' = f_1 F_s' = \left(\frac{1}{4}\right) 1000 = 250\ \text{Hz}$.

$\frac{F_2'}{F_s'} = f_2 \Rightarrow F_2' = \left(\frac{3}{8}\right) 1000 = 375\ \text{Hz}$.

So reconstructed signal is

$$\boxed{x_a'(t) = 3\cos(2\pi \overbrace{(250)}^{F_1'} t) - 2\sin(2\pi \overbrace{(375)}^{F_2'} t)}$$

which is different from input $x_a(t)$, due to aliasing.

5

Postfilter

remove frequency above $F_s/2$.

$F_s = \frac{1}{T} = \frac{1}{5\text{ ms}} = 200\ \text{Hz}$. so $\frac{F_s}{2} = 100\ \text{Hz}$.

looking at $x_a'(t)$, I see both signal components have frequencies $> 100\ \text{Hz}$.

so $\boxed{y_a(t) = 0}$

HW3, problem 1.12 (b)

Statment What is the analog signal we can obtain from $x(n)$ if in the reconstruction process we assume $F_s = 10kHz$?

use Example 1.4.2

$$x_a(t) = 3\cos 100\pi t.$$

Solution

For sampling, use $F_s = 200\,HZ$ as per example 1.4.2, part (b).

so $$x(n) = 3\cos 100\pi\left(n\frac{1}{F_s}\right) = 3\cos\left(100\pi\frac{n}{200}\right)$$

$$\boxed{x(n) = 3\cos\left(2\pi\left(\frac{1}{4}\right)n\right)}$$

For reconstruction, assuming $F_s = 10000$, we set

$$\frac{F'}{F_s} = f_1 \qquad \text{but } f_1 = \frac{1}{4},\ F_s = 10000$$

so $F' = 40000$

so $$y(a) = 3\cos\left(2\pi\,(40000)\,t\right)$$

$$\boxed{y(a) = 3\cos(8000\,\pi t)}$$

✓ 5

HW 3, problem 2.6 (a, b)

Statment

Consider system $y(n) = T[x(n)] = x(n^2)$.

(a) determine if system is time invariant

(b) to clarify the result of part (a), assume signal

$$x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

is applied to the system.

(1) sketch $x(n)$.

(2) Determine and sketch $y(n) = T[x(n)]$

(3) sketch $y_2'(n) = y(n-2)$

(4) Determine and sketch $x_2(n) = x(n-2)$

(5) Determine and sketch $y_2(n) = T[x_2(n)]$

(6) Compare $y_2(n)$ and $y(n-2)$. what is your conclusion?

Solution

(a) a system is time invariant if $x(n) \xrightarrow{T} y(n)$ implies $x(n-k) \xrightarrow{T} y(n-k)$ for every input $x(n)$ and every delay k.

Given system, $y(n) = x(n^2)$ ——— ①

when we delay input by k, we get output

$$\boxed{x(n^2 - k)} \text{ — ②}$$

now, if we delay output by k, then from ①, we get

$$y(n-k) = x((n-k)^2) = \boxed{x(n^2 + k^2 - 2nk)} \text{ — ③}$$

② ≠ ③ For every k ⇒

so $\boxed{\text{NOT Time invariant}}$

✓ 2

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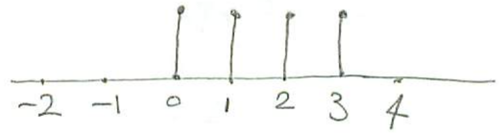
HW 3 problem 2.6 cont

(b)

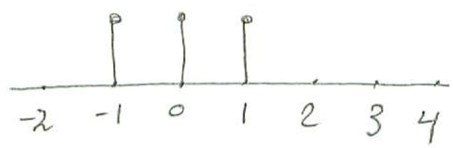
(1) $x(n)$



(2) $y(n) = \mathcal{T}[x(n)]$

n	-2	-1	0	1	2	3	4	5
$x(n)$	0	0	1	1	1	1	0	0
n^2	4	1	0	1	4	9	16	25
$x(n^2)$	0	1	1	1	0	0	0	0

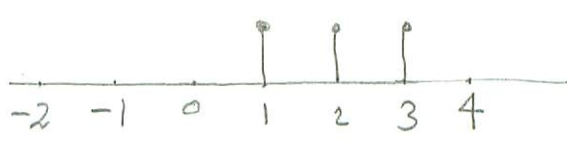
$y(n)$



(3) sketch $y_2'(n) = y(n-2)$

n	-2	-1	0	1	2	3	4	5	6
$y(n)$	0	1	1	1	0	0	0	0	0
$n-2$	-4	-3	-2	-1	0	1	2	3	4
$y(n-2)$	0	0	0	1	1	1	0	0	0

$y_2'(n) = y(n-2)$



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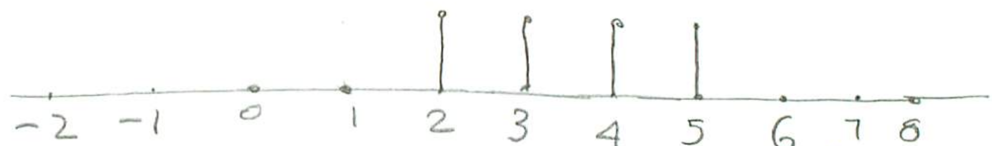
HW problem 2.6 cont

(4) sketch $x_2(n) = x(n-2)$

n	-1	0	1	2	3	4	5	6	7	8
$x(n)$	0	1	1	1	1	0	0	0	0	0
$n-2$	-3	-2	-1	0	1	2	3	4	5	6
$x(n-2)$	0	0	0	1	1	1	1	0	0	0

$x_2 = x(n-2)$

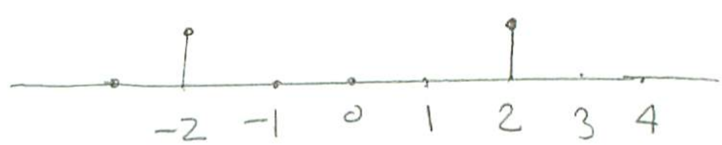


(5) Determin, sketch $y_2(n) = T[x_2(n)]$

$T[x_2(n)] = x_2(n^2)$

n	-2	-1	0	1	2	3	4	5	6
$x_2(n)$	0	0	0	0	1	1	1	1	0
n^2	4	1	0	1	4	9	16	25	36
$x_2(n^2)$	1	0	0	0	1	0	0	0	0

$y_2(n) = T[x_2(n)]$



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(6) $y_2(n)$ is output due to $x_2(n)$ which is a delayed input by $K=2$

$y(n-2)$ is delayed output by $K=2$ due to same input $x(n)$.

since looking at both signals shows that they are different (part (5) $\neq$ part (3)) $\Rightarrow$ system is $\boxed{\text{NOT time invariant}}$

HW#3 , problem 2.7 a, b, c, d.

Problem

A discrete system can be

(1) static or dynamic
(2) Linear or non linear
(3) Time variant or invariant
(4) Causal or non causal
(5) stable or non stable.

determine the above for the following systems

(a) $y(n) = \cos(x(n))$

(b) $y(n) = \sum_{k=-\infty}^{\infty} x(k)$

(c) $y(n) = x(n)\cos(\omega_0 n)$

(d) $y(n) = x(-n+2)$

Answer

(a)

(1) $\boxed{\text{static}}$ ✓ since memoryless (does not depend on past or future)

(2) $T[a x_1(n) + b x_2(n)] = \cos(a x_1(n) + b x_2(n)) = \cos(a x_1(n))\cos(b x_2(n)) - \sin(a x_1(n))\sin(b x_2(n))$

$aT[x_1(n)] + bT[x_2(n)] = a\cos(x_1(n)) + b\cos(x_2(n)) \Rightarrow \boxed{\text{NOT Linear}}$ ✓

(3) delayed input gives output $= \cos(x(n-k))$

but delayed output $y(n-k) = \cos(x(n-k))$ $\Big\} \Rightarrow \boxed{\text{Time invariant}}$ ✓

(4) $\boxed{\text{Causal}}$ since output does not depend on future input. ✓

(5) Select bounded input signal $x(n) = c\,\delta(n)$

where C is constant. then output

$y(0) = \cos(x(0)) = \cos(c)$
$y(1) = \cos(c\,\delta(1)) = \cos(0) = 1$
$y(2) = \cos(c\,\delta(2)) = \cos(0) = 1$
...

since $\cos \leq 1 \Rightarrow$ bounded output. $\Rightarrow \boxed{\text{stable}}$ BIBO ✓

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→

(b)

(1) $\boxed{\text{dynamic.}}$ since requires memory. output does not only depend on current input.

(2) $\boxed{\text{Linear}}$

(3) $\boxed{\text{time Invariant}}$ since for each output at any time depend on all past and all future input.

(4) $\boxed{\text{not Causal}}$ since output depends on future values.

(5) for bounded input $x(n) = C\delta(n)$, where C is constant, (unit sample seq.)

$$y(n) = \cdots + C\delta(-2) + C\delta(-1) + C\delta(0) + C\delta(1) + C\delta(2) + \cdots$$

$$= \cdot 0 + 0 \quad 0 + C(1) + 0 + 0 + \cdots$$

$$= C$$

so $y(n) = C$. so $y(n)$ is bounded since C is some constant $< \infty$.

hence $\boxed{\text{Stable}}$ BIBO

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(c)

$$y(n) = x(n)\cos(\omega_0 n)$$

(1) $\boxed{\text{static}}$ since depend on on current input.

(2) $T[a\,x_1(n) + b x_2(n)] = [a_1 x_1(n) + b\,x_2(n)]\cos(\omega_0 n)$ ——— ①

$aT[x_1(n)] + bT[x_2(n)] = a\,x_1(n)\cos(\omega_0 n) + b\,x_2(n)\cos(\omega_0 n)$ ——— ②

so ① = ② $\Rightarrow$ $\boxed{\text{Linear}}$

(3) delayed input gives $y(n,k) = x(n-k)\cos(\omega_0 n)$

delayed output gives $y(n-k) = x(n-k)\cos(\omega_0(n-k))$

so $y(n,k) \neq y(n-k)$ for all k.

hence $\boxed{\text{NOT time invariant}}$

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(4) $\boxed{\text{Causal}}$ since do not depend on future

(5) apply $C\delta(n)$ as input $\Rightarrow y(n) = C\delta(n)\cos(\omega_0 n)$

$\Rightarrow$ bounded output since $\cos \le |1|$. C constant $\Rightarrow$ $\boxed{\text{Stable}}$ BIBO

HW #3, problem 2.7 Cont

(d) $y(n) = x(-n+2)$

(1) Static. since depend only on current input. X

(2) $T[a x_1(n) + b x_2(n)] = a x_1(-n+2) + b x_2(-n+2)$

$aT[x_1(n)] + b T[x_2(n)] = a x_1(-n+2) + b x_2(-n+2)$ $\Rightarrow$ $\boxed{\text{Linear}}$ ✓

(3) a delayed input gives

$$y(n,k) = x(-n+2-k)$$

a delayed output is $y(n-k) = x(-(n-k)+2) = x(-n+2+k)$

so $y(n,k) \neq y(n-k) \Longrightarrow$ $\boxed{\text{NOT time invariant}}$ ✓

(4) for $n=0$ we set

$$y(0) = x(2).$$

hence $y(0)$ depends on future input. $\Rightarrow$ $\boxed{\text{NOT Causal}}$ ✓

(5) supply input $x(n) = C\,\delta(n)$.

So output $y(n) = C\,\delta(-n+2)$

4

so
$$\begin{aligned} &\vdots \\ y(-1) &= 0 \\ y(0) &= 0 \\ y(1) &= 0 \\ y(2) &= C\delta(0) = C \\ y(3) &= 0 \\ &\vdots \end{aligned}$$

So BIBO $\boxed{\text{stable}}$ since C is constant.

✓

HW # 3 problem 2.10

problem The following input-output pairs have been observed during the operation of a time invariant system:

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{0, \underset{\uparrow}{1}, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw any conclusions regarding the linearity of the system? what is the impulse response of the system?

Answer

a system is Linear if any input $x(n)$ convolve with $h(n)$ will give the output from $x(n)$.

looking at $x_3(n)$, we see it is $\delta(n-3)$.

since this is time invariant, then $y_3(n)$ is the same as $h(n-3)$.

i.e a delayed input gives a delayed output for time invariant.

so, to find $h(n)$, we shift $x_3(n)$ to left by 3, and this gives $\delta(n)$. so $h(n)$ is shifted $y_3(n)$ to left by 3 as well.

so $\boxed{h(n) = \{1\ 2\ 1\ 0\ \underset{\uparrow}{0}\}}$

so $H(z) = z^2 + 2z^3 + z^4$

now, looking at $x_1(n)$, $\Rightarrow X_1(z) = 1 + 2z^{-2}$.

so $Y_1(z) = H(z)X_1(z) = z^4 + 2z^3 + 3z^2 + 4z + 2$

so $y_1(n) = \{1, 2, 3, 4, \underset{\uparrow}{2}\}$ which is not $y_1(n)$.

$\Rightarrow$ system not Linear

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HW# 3 problem 2.16

statment

(a) If $y(n) = x(n) * h(n)$, show that $\Sigma_y = \Sigma_x \Sigma_h$ where

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x(n).$$

(b) compute convolution $y(n) = x(n) * h(n)$ of the following and check correctness by using test in (a)

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

(2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

(3) $x(n) = \{0, 1, -2, 3, -4\}$ $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

(4) $x(n) = \{1, 2, 3, 4, 5\}$ $h(n) = \{1\}$

(5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$ $h(n) = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$

Solution

(a)

$$\Sigma_y = \sum_{n=-\infty}^{\infty} y(n)$$

$$\Sigma_x = \sum_{n=-\infty}^{\infty} x(n)$$

$$\Sigma_h = \sum_{n=-\infty}^{\infty} h(n)$$

so $$\Sigma_x \Sigma_h = \left[\sum_{n=-\infty}^{\infty} x(n)\right]\left[\sum_{n=-\infty}^{\infty} h(n)\right]$$

$$= \left[\cdots + x(-1) + x(0) + x(1) + \cdots\right]\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right]$$

$$\begin{aligned} = \cdots &+ x(-1)\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right] \\ &+ x(0)\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right] \\ &+ x(1)\left[\cdots + h(-1) + h(0) + h(1) + \cdots\right] + \\ &\cdots \end{aligned}$$

$$\begin{aligned} = \cdots &+ \left[\cdots + x(-1)h(-1) + x(-1)h(0) + x(-1)h(1) + \cdots\right] \\ &+ \left[\cdots + x(0)h(-1) + x(0)h(0) + x(0)h(1) + \cdots\right] \qquad (1) \\ &+ \left[\cdots + x(1)h(-1) + x(1)h(0) + x(1)h(1) + \cdots\right] \\ &\cdots \end{aligned}$$

✓

but since $y(n) = x(n) * h(n)$

then $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

so
$$\left.\begin{aligned} y(0) &= \cdots + x(-1)h(1) + x(0)h(0) + x(1)h(-1) + \cdots \\ y(1) &= \cdots + x(-1)h(2) + x(0)h(1) + x(1)h(0) + \cdots \\ y(2) &= \cdots + x(-1)h(3) + x(0)h(2) + x(1)h(1) + \cdots \end{aligned}\right\} \text{ — } ②$$

Looking at ①, we see it is the same as ②

for example $\boxed{y(0)}$ inside ② can be seen inside ① as follows

①: $\cdots \left[\cdots + x(-1)h(-1) + x(-1)h(0) + \boxed{x(-1)h(1)} + \cdots\right]$
$+ \left[\cdots + x(0)h(-1) + \boxed{x(0)h(0)} + x(0)h(1) + \cdots\right]$
$+ \left[\cdots + \boxed{x(1)h(-1)} + x(1)h(0) + x(1)h(1) + \cdots\right]$
$\cdots$

✓

similarly $y(1)$ is the diagonal to the right of the above diagonal., and $y(2)$, is the diagonal to the right of that, etc...

so $$\boxed{\sum_x \sum_h = \sum_y}$$

✓ 5

⑥ (1) find convolution $x(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

n	k	-1	0	1	2	3	4	
	$x(k)$	0	1	2	4	0	0	$y(n)$
0	$h(0-k)$	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	$h(-2)$ 0	$h(-3)$ 0	$h(-4)$ 0	1
1	$h(1-k)$	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	$h(-2)$ 0	$h(-3)$ 0	3
2	$h(2-k)$	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	$h(-2)$ 0	7
3	$h(3-k)$	$h(4)$ 1	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	$h(-1)$ 0	7
4	$h(4-k)$	$h(5)$ 0	$h(4)$ 1	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	$h(0)$ 1	7
5	$h(5-k)$	$h(6)$ 0	$h(5)$ 0	$h(4)$ 1	$h(3)$ 1	$h(2)$ 1	$h(1)$ 1	6

→

$x(k)$	$h(6-k)$	$h(7)$	$h(6)$	$h(5)$	$h(4)$	$h(3)$	$h(2)$	$h(1)$	$h(0)$	$h(-1)$	$y(n)$
		0	1	2	4	0	0	0	0	0	
6		0	0	0	1	1	1	1	1	0	4
7		0	0	0	0	1	1	1	1	1	0
8		0	0	0	0	1	1	1	1	1	0

so
$$\boxed{y(n) = \{\ldots, 0, 0, \underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4, 0, 0, \ldots\}}$$

$$\Sigma_y = 1+3+7+7+7+6+4 = 35$$

same. so test ⓐ verified.

$$\left.\begin{array}{l} \Sigma_x = 1+2+4 = 7 \\ \Sigma_h = 1+1+1+1+1 = 5 \end{array}\right\} \Rightarrow \text{multiply} \Rightarrow 35$$

② →

(2) $x(n) = \{\underset{\uparrow}{1}, 2, -1\}$, $h(n) = \{\underset{\uparrow}{1}, 2, -1\}$

n	k	-1	0	1	2	3	4	y(n)
	x(k)	0	1	2	-1	0	0	
-1	h(-1-k)	h(0) 1	h(-1) 0	h(-2) 0	h(-3) 0	h(-4) 0	h(-5) 0	0
0	h(0-k)	2	1	0	0	0	0	1
1	h(1-k)	-1	2	1	0	0	0	4
2	h(2-k)	0	-1	2	1	0	0	2
3	h(3-k)	0	0	-1	2	1	0	-4
4	h(4-k)	0	0	0	-1	2	1	2
5	h(5-k)	0	0	0	0	-1	2	0
6	h(6-k)	0	0	0	0	0	-1	0

so $\boxed{y(n) = \{0, 0, \underset{\uparrow}{1}, 4, 2, -4, 1, 0, 0, \ldots\}}$

$\Sigma_y = 1 + 4 + 2 - 4 + 1 = 4$ same. test (a) verified.

$\left.\begin{array}{l} \Sigma_x = 1 + 2 - 1 = 2 \\ \Sigma_h = 1 + 2 - 1 = 2 \end{array}\right\}$ multiply $= 4$

✓

v

(3) $x(n) = \{ \underset{\uparrow}{0}, 1, -2, 3, -4, 0 \}$ $h(n) = \{ \underset{\uparrow}{\tfrac{1}{2}}, \tfrac{1}{2}, 1, \tfrac{1}{2} \}$

n	k	0	1	2	3	4	5	y(n)
	x(k)	0	1	-2	3	-4	0	
0	x(0-k)	h(0) $\frac{1}{2}$	h(-1) 0	0	0	0	0	0
1	x(1-k)	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	$\frac{1}{2}$
2	x(2-k)	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2} - \frac{1}{2}2 = -\frac{1}{2}$
3	x(3-k)	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$1 - 2\frac{1}{2} + \frac{3}{2} = 1 - 1 + \frac{3}{2} = \frac{3}{2}$
4	x(4-k)	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2} - 2 + \frac{3}{2} - \frac{4}{2} = -2$
5	x(5-k)	0	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}\times 2 + 3 - \frac{4}{2} = 2 - 2 = 0$
6	x(6-k)	0	0	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{3}{2} - 4 = -2\frac{1}{2}$
7		0	0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2} \times 4 = -2$

so $\boxed{y(n) = \{ \ldots, 0, \underset{\uparrow}{0}, \tfrac{1}{2}, -\tfrac{1}{2}, 1\tfrac{1}{2}, -2, 0, -2\tfrac{1}{2}, -2, 0, 0, \ldots \}}$

$\sum_y = -5$ same. verified using test (a)

$\left.\begin{array}{l} \sum_x = -2 \\ \sum_h = 2\frac{1}{2} \end{array}\right\}$ multiply $= -2 \times \frac{5}{2} = -5$

✓

2

(4) $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ $h(n) = \{\underset{\uparrow}{1}\}$

n	k	0	1	2	3	4	5	6
	x(k)	1	2	3	4	5	0	y(n)
0	h(0-k)	1	0	0	0	0	0	1
1	h(1-k)	0	1	0	0	0	0	2
2	h(2-k)	0	0	1	0	0	0	3
3	h(3-k)	0	0	0	1	0	0	4
4	h(4-k)	0	0	0	0	1	0	5
5	h(5-k)	0	0	0	0	0	1	0

$$\Rightarrow \boxed{y(n) = \{\ldots, 0, 0, \underset{\uparrow}{1}, 2, 3, 4, 5, 0, 0 \ldots\}}$$

$\sum_y = 15$ verified same.

$\left.\begin{array}{l}\sum_x = 15 \\ \sum_h = 1\end{array}\right\}$ $15 \times 1 = 15$

2

(5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$ $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

n	k	0	1	2	3	4	5	
	$x(k)$	1	-2	3	0	0	0	$y(n)$
0	$h(0-k)$	0	0	0	0	0	0	0
1	$h(1-k)$	0	0	0	0	0	0	0
2	$h(2-k)$	1	0	0	0	0	0	1
3	$h(3-k)$	1	1	0	0	0	0	-1
4	$h(4-k)$	1	1	1	0	0	0	2
5	$h(5-k)$	1	1	1	1	0	0	2
6	$h(6-k)$	0	1	1	1	1	0	1
7	$h(7-k)$	0	0	1	1	1	1	3

$$\Rightarrow \boxed{y(n) = \{\ldots 0, 0, 1, -1, 2, 2, 1, 3, 0, \ldots\}}$$

✓

$\Sigma_y = 8$ verified same

$\left.\begin{array}{l}\Sigma_x = 2 \\ \Sigma_h = 4\end{array}\right\}$ $2 \times 4 = 8$

2

HW #3, problem 2.23

discrete time system $y(n) = n\,y(n-1) + x(n)$ $n \geq 0$ is at rest $(y(-1)=0)$. Check if system is Linear time invariant and BIBO

Solution

Since system is relaxed, need only to consider Linearity for relaxed system.

$$y(0) = 0 + x(0) = x(0)$$
$$y(1) = 1\cdot y(0) + x(1) = 1\cdot x(0) + x(1)$$
$$y(2) = 2\cdot y(1) + x(2) = 2\cdot[x(0)+x(1)] + x(2)$$
$$y(3) = 3\cdot y(2) + x(3) = 3\left[2[x(0)+x(1)] + x(2)\right] + x(3)$$
$$= 3\cdot 2\,[x(0)+x(1)] + 3\cdot x(2) + x(3)$$
$$y(4) = 4\cdot y(3) + x(4) = 4\left[3\cdot 2\,[x(0)+x(1)] + 3\cdot x(2) + x(3)\right] + x(4)$$
$$= 4\cdot 3\cdot 2\,[x(0)+x(1)] + 4\cdot 3\,x(2) + 4\,x(3) + x(4)$$

so
$$y(n) = n!\,x(0) + n!\,x(1) + \frac{n!}{2!}x(2) + \frac{n!}{3!}x(3) + \frac{n!}{4!}x(4) + \cdots$$
$$= n!\left[\frac{x(0)}{0!} + \frac{x(1)}{1!} + \frac{1}{2!}x(2) + \frac{1}{3!}x(3) + \cdots\right]$$

$$\boxed{y(n) = n! \sum_{m=0}^{n} \frac{x(m)}{m!}}$$

$\longrightarrow$

$$T\left[a x_1(n) + b x_2(n)\right] = n! \sum_{m=0}^{n} \frac{a x_1(m) + b x_2(m)}{m!}$$

$$= n! \left(\sum_{m=0}^{n} \frac{a x_1(m)}{m!} + \frac{b x_2(m)}{m!} \right)$$

$$= n! \left(\sum_{m=0}^{n} \frac{a x_1(m)}{m!} + \sum_{m=0}^{n} \frac{b x_2(m)}{m!} \right)$$

$$= n! \sum_{m=0}^{n} \frac{a x_1(m)}{m!} + n! \sum_{m=0}^{n} \frac{b x_2(m)}{m!}$$

$$= a T\left[x_1(n)\right] + b T\left[x_2(n)\right]$$

$\Rightarrow$ $\boxed{\text{Linear}}$ ✓

To check for time invariant.

a delayed input produces output

$$y(x, k) = n! \sum_{m=0}^{n} \frac{x(m-k)}{m!}$$

a delayed output is $y(x-k) = (n-k)! \sum_{m=0}^{n-k} \frac{x(m)}{m!}$

to see if $y(x,k) = y(x-k)$, $\boxed{\text{try} \quad n=3, \ k=1}$

$$y(x,k) = 3! \sum_{m=0}^{3} \frac{x(m-1)}{m!} = 3! \left[\frac{\overset{=0}{\cancel{x(-1)}}}{1} + \frac{x(0)}{1} + \frac{x(1)}{2!} + \frac{x(2)}{3!} \right]$$

$$= \boxed{3! \left[x(0) + \frac{x(1)}{2!} + \frac{x(2)}{3!} \right]} \quad \text{—— (1)}$$

$$y(x-k) = 2! \sum_{m=0}^{2} \frac{x(m)}{m!} = 2! \left[x(0) + \frac{x(1)}{1} + \frac{x(2)}{2!} \right] \quad \text{—— (2)}$$

we see that $(1) \neq (2)$ $\Rightarrow$ $\boxed{\text{NOT time invariant}}$ ✓

$\rightarrow$

to check for BIBO stable ;

give the system input $c\,\delta(n)$ and see if output $y(n)$ is bounded.

$$y(n) = n! \sum_{m=0}^{n} \frac{c\,\delta(m)}{m!}$$

$$= n! \left[c\,\delta(0) + c\,\delta(1) + c\,\frac{\delta(2)}{2!} + \cdots \right]$$

since $\delta(n) = 0$ for all $n \neq 0$, than

$$y(n) = n! \, [\, c \,]$$

$$\boxed{y(n) = c\, n!}$$ where c is a constant.

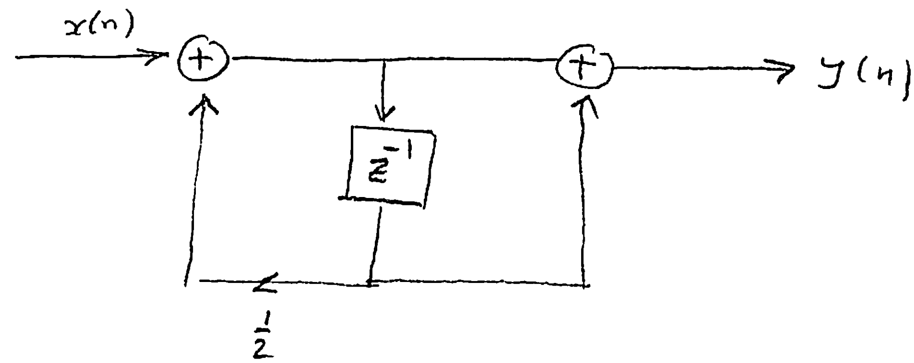
since $n!$ grows with no limit as n grows, so for a bounded input $\delta(n)$ we obtain unbounded output.

so $\boxed{\text{NOT stable}}$

8

HW #3 problem 2.44

Consider system



(a) Compute first 10 samples of its impulse response.

(b) Find input-output relation.

(c) Apply input $x(n) = \{ \underset{\uparrow}{1}, 1, 1, \dots \}$ and compute the first 10 samples of the output.

(d) Compute the first 10 samples of output for input in part (c) by using convolution.

(e) Is system causal? Stable?

Solution

(a) $\boxed{y(n) = \frac{1}{2} y(n-1) + x(n)}$

$y(n) = x(n) + x(n-1) + \frac{1}{2} y(n-1)$

Assume relaxed system. so $y(-1) = 0$

1

$$y(0) = 0 + x(0) = x(0)$$

$$y(1) = \frac{1}{2} y(0) + x(1) = \frac{1}{2} x(0) + x(1)$$

$$y(2) = \frac{1}{2} y(1) + x(2) = \frac{1}{2}\left(\frac{1}{2} x(0) + x(1)\right) + x(2)$$

$$= \frac{1}{4} x(0) + \frac{1}{2} x(1) + x(2)$$

$$y(3) = \frac{1}{2} y(2) + x(3) = \frac{1}{2}\left\{\frac{1}{4} x(0) + \frac{1}{2} x(1) + x(2)\right] + x(3)$$

$$= \frac{1}{8} x(0) + \frac{1}{4} x(1) + \frac{1}{2} x(2) + x(3)$$

so $\boxed{y(n) = \sum_{K=0}^{n} \frac{1}{2^K} x(n-K)}$

when $x(n) = \delta(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$

then $y(0) = \delta(0) = 1$

$y(1) = \frac{1}{1}x(1-0) + \frac{1}{2}x(1-1) = x(1) + \frac{1}{2}x(0) = \frac{1}{2}$

$y(2) = \frac{1}{1}x(2-0) + \frac{1}{2}x(2-1) + \frac{1}{4}x(2-2) = x(2) + \frac{1}{2}x(1) + \frac{1}{4}x(0) = \frac{1}{4}$

$y(3) = \frac{1}{8}$, $y(4) = \frac{1}{16}$, $y(5) = \frac{1}{32}$, $y(6) = \frac{1}{64}$

$y(7) = \frac{1}{128}$, $y(8) = \frac{1}{256}$, $y(9) = \frac{1}{512}$.

1

so $\boxed{y = \{\underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}\}}$

is the response to impulse. i.e $\boxed{y(n) = \left(\frac{1}{2}\right)^n}$ $n \geq 0$

(b) h is given by part (a). since $h(n)$ is the impulse response.

So $\boxed{h(n) = \left(\frac{1}{2}\right)^n u(n)}$

(c) using difference equation

$y(0) = 0 + x(0) = x(0) = 1$

$y(1) = \frac{1}{2}y(0) + x(1) = \frac{1}{2} + 1 = \frac{3}{2}$

2

$y(2) = \frac{1}{2}\left(\frac{3}{2}\right) + x(2) = \frac{1}{2}\left(\frac{3}{2}\right) + 1 = \frac{3}{4} + 1 = \frac{7}{4}$

$y(3) = \frac{1}{2}y(2) + x(3) = \frac{1}{2}\left[\frac{7}{4}\right] + 1 = \frac{7}{8} + 1 = \frac{15}{8}$

$y(4) = \frac{1}{2}\left(\frac{15}{8}\right) + 1 = \frac{15}{16} + 1 = \frac{31}{16}$

$y(5) = \frac{1}{2}\left(\frac{31}{16}\right) + 1 = \frac{31}{32} + 1 = \frac{63}{32}$

$y(6) = \frac{1}{2}\frac{63}{32} + 1 = \frac{63}{64} + 1 = \frac{127}{64}$

$y(7) = \frac{255}{128}$, $y(8) = \frac{511}{256}$, $y(9) = \frac{1023}{512}$

so $y = \{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \frac{63}{32}, \frac{127}{64}, \frac{255}{128}, \frac{511}{256}, \frac{1023}{512}, \ldots\} \rightarrow$

so $y(n) = \dfrac{(2 \cdot 2^n) - 1}{2^n} = \boxed{\dfrac{2^{n+1} - 1}{2^n}}$

(d) Now find $y(n)$ again using convolution

$x(n) = \{1, 1, 1, \cdots\}$ $\quad h(n) = (\tfrac{1}{2})^n u(n) = \{1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16}, \cdots\}$

n	K	0	1	2	3	4	5	6	7	8	9	$y(n)$
	$x(k)$	1	1	1	1	1	1	1	1	1	1	
0	$h(0-k)$	$h(0)$ 1	$h(-1)$ 0	0	0	0	0	0	0	0	0	1
1	$h(1-k)$	$\frac{1}{2}$	1	0	0	0	0	0	0	0	0	$\frac{1}{2}+1 = \frac{3}{2}$
2	$h(2-k)$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	0	0	0	0	$\frac{1}{4}+\frac{1}{2}+1 = \frac{7}{4}$
3	$h(3-k)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	0	0	0	$\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+1 = \frac{15}{8}$
4	$h(4-k)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	0	0	$\frac{31}{16}$
5	$h(5-k)$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	0	0	0	$\frac{63}{32}$
6	$h(6-k)$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	0	$\frac{127}{64}$
7	$h(7-k)$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	0	$\frac{255}{128}$
8	$h(8-k)$	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	0	$\frac{511}{256}$
9	$h(9-k)$	$\frac{1}{512}$	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1023}{512}$

hence $y(n) = \{1, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \cdots\}$ same as found in part (c)

(e) system is stable. by ratio test $y(n)$ converges.. i.e $\left| \dfrac{(\frac{1}{2})^{n+1}}{(\frac{1}{2})^n} \right| \to \frac{1}{2}$

as response to impulse.

so $\boxed{\text{BIBO stable}}$

2

Since $y(n) = \frac{1}{2} y(n-1) + x(n)$, we see that $y(n)$ do not depend on future n. $\Rightarrow$ $\boxed{\text{Causal}}$