EECS152A: HOMEWORK #2

Due: October 12, 2004

Problems from the textbook: 1.7, 1.8, 1.9, 1.10(a)(b)(c)

HW#2

EECS 152A, Digital Signal processing

UCI. Fall 2004

By Nasser Abbasi

HW#2

Problem 1.7

12/15

Statment An analog Signal contains frequencies up to 10 KHZ.

(a) what range of sampling Frequencies allow exact reconstruction of this signal From its samples?

(b) suppose we sample with $F_s = 8$ KHZ. explain what happens to frequency $F_1 = 5$ KHZ.

(c) repeat (b) For frequency $F_2 = 9$ KHZ.

Answer

(a) $F_s > 2F_{max}$. where $F_{max} = 10$ KHz. (given).

so $\boxed{F_s > 20 \text{KHZ}}$ allow exact reconstruction.

(b) with $F_s = 8$ KHZ, Folding Frequency is $\frac{F_s}{2} = 4$ KHZ.

since $F_1 > \frac{F_s}{2}$ then F_1 will not be recovered but will alias to a frequency < 4 KHZ. F_1 will alias to a Frequency $\boxed{F_1 + KF_s}$ where K is ±1 or ±2 or ±3 etc....

Folding Frequency = 4 KHZ

so need to find K such that $F_1 + KF_s < |4 \text{KHZ}|$.

3

so with $\boxed{K=-1}$ we set alias Frequency $= 5 + (-1)8 = \boxed{-3 \text{KHZ}}$

hence $\boxed{F_1 \text{ will alias to } -3 \text{KHZ}}$

(c) here $F_2 = 9$ KHZ. so need to Find K: $9 + K(8) < |4|$

so $K = -1$.

so $\boxed{F_2 \text{ will alias to } 9 - 8 = 1 \text{ KHZ}}$

HW#2

Problem 1.8

Statment

An Analog ECG signal contains useful frequencies up to 100 HZ.

(a) what is the Nyquist rate for this signal?

(b) Suppose we sample this signal at rate 250 samples/sec, what is highest frequency that can be represented uniquely at this sampling rate?

Answer

(a) Nyquist rate $= 2 F_{max}$

$= 2 (100 \text{ Hz}) = \boxed{200 \text{ Hz}}$ ✓

(b) $F_s = 250$ sample/sec. (or 250 Hz)

so highest Frequency that can be sampled uniquely is $\boxed{\frac{F_s}{2}} = \boxed{125 \text{ Hz}}$ ✓

Folding Frequency

2

HW#2
Problem 1.9

statement

An analog signal $x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$ is sampled at rate 600 times per second.
(a) Find Nyquist sampling rate for $x_a(t)$.
(b) Find Folding Frequency.
(c) what are the frequencies in radians in resulting discrete time signal $x(n)$?
(d) if $x(n)$ is passed through an ideal D/A convertor, what is the reconstructed signal $y_a(t)$?

Answer

(a) F_{max} : $2\pi F_{max} t \equiv 720\pi t$

2

so $F_{max} = \frac{720}{2} = 360$ Hz.

so Nyquist rate $= 2F_{max} = \boxed{720 \text{ Hz}}$

(b) Folding frequency $= \frac{F_s}{2} = \frac{600}{2} = \boxed{300 \text{ Hz}}$

(c) $x(n) = \sin(480\pi(nT)) + 3\sin(720\pi(nT))$

$= \sin(480\pi \frac{n}{F_s}) + 3\sin(720\pi \frac{n}{F_s})$

$= \sin(\frac{480}{600}\pi n) + 3\sin(\frac{720}{600}\pi n)$

so $2\pi f_1 n = \frac{480}{600}\pi n \Rightarrow f_1 = \frac{240}{600} = \frac{6}{15} = \frac{2}{5}$ samples/sec (ok $< \frac{1}{2}$)

but $\omega_1 = 2\pi f_1 \Rightarrow \omega_1 = 2\pi \frac{2}{5} = \boxed{\frac{4}{5}\pi}$

to find ω_2:

$2\pi f_2 n = \frac{720}{600}\pi n \Rightarrow f_2 = \frac{360}{600} = \frac{18}{30} = \frac{9}{15} = \frac{3}{5}$ Samp/sec

2

$\frac{3}{5} > \frac{1}{2} \Rightarrow \boxed{f_2 \Rightarrow -\frac{2}{5}}$ so $\omega_2 = 2\pi \frac{-2}{5} = \boxed{\frac{-4}{5}\pi}$

HW# 2
Problem 1.9 (cont.)

(d) $$y_a(t) = \sum_{n=-\infty}^{n=+\infty} x_a\left(\frac{n}{F_s}\right) \operatorname{sinc}\left(2\pi F_{max}\left(t - \frac{n}{F_s}\right)\right)$$

where $F_s = 600$ Hz.
$F_{max} = 360$ Hz.

so $$\boxed{y_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{600}\right) \operatorname{sinc}\left(2\pi(360)\left(t - \frac{n}{600}\right)\right)}$$

this is called sinc interpolation. ~~0~~ 2

Refer to the sol^n

Here, $t = 2/5$.
$f = f F_s = (2/5)600 = 240$.

$$y_a(t) = -2\sin[2\pi(F)f]t.$$
$$= -2\sin[2\pi(240)2/5]t.$$
$$= -2\sin(480\pi)t \quad //.$$

HW #2

Problem 1.10

statment

A digital communication link carries binary-coded words representing samples of an input signal

$$X_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

the Link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels.

(a) what is the sampling frequency and the folding frequency?

(b) what is the Nyquist rate for $X_a(t)$?

(c) what are the frequencies in resulting discrete signal $x(n)$?

Answer. Each i/p sample = 10bits, since $1024 = 2^{10}$.

$f_s = 1000$

(a) Consider each bit as sample. then $\boxed{F_s = 10{,}000 \text{ sample/sec}}$

so Folding frequency $= \frac{F_s}{2} = \boxed{5{,}000 \text{ Hz}}$ $f_{fold} = 500$

(b) Find F_{max}

(1) $1800\pi t = 2\pi F_{max} t \Rightarrow F_{max} = \frac{1800}{2} = \boxed{900 \text{ Hz}}$

so Nyquist frequency $= 2 F_{max} = \boxed{1800 \text{ Hz}}$ ✓

(c) $F_1 = 300$ Hz, $F_2 = 900$ Hz.

Folding frequency = 5000 Hz. so no aliasing since Folding frequency > F_{max}.

$$x(n) = 3\cos 600\pi\left(\frac{n}{F_s}\right) + 2\cos 1800\pi\left(\frac{n}{F_s}\right)$$

$$= 3\cos 600\pi\left(\frac{n}{10000}\right) + 2\cos 1800\pi\left(\frac{n}{10000}\right)$$

$$= 3\cos\frac{3}{50}n\pi + 2\cos\frac{9}{50}n\pi$$

$f_1 = \frac{3}{10}$; $f_2 = \frac{9}{10}$ or $\frac{1}{10}$

$\Rightarrow \boxed{f_1 = \left(\frac{3}{100}\right) \text{ Samples/sec}}$, $\boxed{f_2 = \left(\frac{9}{100}\right) \text{ sample/sec}}$