HW#1

EECS 152A, Digital Signal processing

UCI. Fall 2004

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EECS 152 DSP

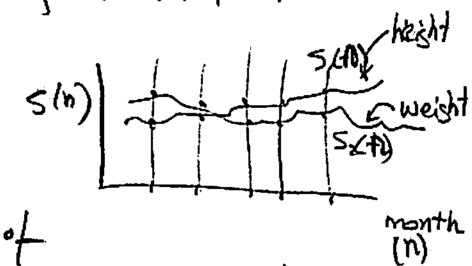
HW#1

9/10

Problem 1.1 (e)

Statment: Classify the following signal according to whether they are (1) one or multidimensional (2) single or multichannel, (3) continous time or discrete time, (4) analog or digital (in amplitude). Give breif explanation.

(e) weight and height measurments of a child taken every month.



Answor

(1) this is $\boxed{\text{one dimension}}$ since function of one independent variable (the month in this case).

(2) ~~this~~ is $\boxed{\text{multichannal}}$. one channel is the height signal, and the second channel is the weight.

1

(3) This is $\boxed{\text{discrete time}}$. since time here is month number.

(4) This is $\boxed{\text{analog}}$. because weight and height are continous quantities.

$$\boxed{s(n) = \begin{bmatrix} s_1(n) \\ s_2(n) \end{bmatrix}}$$

EECS 152 DSP

HW#1

Problem 1.2

Statment

Determine which of following sinusoids are periodic and Compute Fundemental period

(a) $\cos 0.01\pi n$ (b) $\cos\left(\pi \frac{30n}{105}\right)$ (c) $\cos 3\pi n$ (d) $\sin 3n$

(e) $\sin\left(\pi \frac{62n}{10}\right)$

Answer

General Form of discrete sinusoidal signal is

$$A \sin(2\pi f n + \theta) \quad \text{or} \quad A \cos(2\pi f n + \theta).$$

where A is amplitude, f is Cycles per Sample, θ is phase in radians, n is sample number.

discrete-time sinusoid is periodic only if f is rational.

(a) $\cos 0.01\pi n = \cos(2\pi f n + \theta)$

$\Rightarrow 0.1\pi n = 2\pi f n \Rightarrow f = \frac{0.1}{2}$ cycles per sample.

so f is rational $\Rightarrow$ $\boxed{\text{periodic}}$ ✓

4) To Find Fundemental period. write $f = \frac{K}{N}$ where K, N are relative primes. then N is the Fundemental period.

so $f = \frac{1}{20} \Rightarrow$ $\boxed{\text{F. Period} = 20}$ samples

→

(b) $\cos\left(\pi \frac{30n}{105}\right)$.

write $\cos\left(\pi \frac{30n}{105}\right) = \cos(2\pi f n + \theta)$

so $\pi \frac{30n}{105} = 2\pi f n \Rightarrow f = \frac{15}{105} \Rightarrow \boxed{\text{periodic}}$ since rational

$\frac{15}{105} = \frac{3}{21} = \frac{1}{7} \Rightarrow \boxed{\text{Fund. periode} = 7}$ samples.

(c) $\cos 3\pi n$

write as $\cos 3\pi n = \cos(2\pi f n + \theta)$

so $3\pi n = 2\pi f n \Rightarrow f = \frac{2}{3} \cdot 3/2 \Rightarrow \boxed{\text{periodic}}$ since rational.

since $\frac{2}{3}$ allready realtinly prime $\Rightarrow \boxed{\text{Fund period} = 3 \text{ samples}}$

(d) $\sin 3n$

$\sin 3n = \sin(2\pi f n + \theta) \Rightarrow 3n = 2\pi f n \Rightarrow f = \frac{3}{2\pi}$

$\Rightarrow \boxed{\text{NOT periodic}}$ since not rational.

(e) $\sin\left(\pi \frac{62n}{10}\right)$

$\sin\left(\pi \frac{62n}{10}\right) = \sin(2\pi f n + \theta) \Rightarrow 2\pi f n = \frac{62 n \pi}{10}$

$f = \frac{62}{20} \Rightarrow \boxed{\text{periodic}}$

$\frac{62}{20} = \frac{31}{10} \Rightarrow$ Fundemental period $= \boxed{10 \text{ samples}}$

1.5

Problem statment

Consider signal $X_a(t) = 3\sin(100\pi t)$

(a) sketch signal $x_a(t)$ for $0 \le t \le 30$ ms

(b) signal $x_a(t)$ is sampled with $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT)$, $T = \frac{1}{F_s}$ and show that it is periodic.

(c) Compute sample values in one period of $x(n)$. Sketch $x(n)$ on same diagram with $X_a(t)$. What is period of the discrete-time signal in milliseconds?

(d) Can you Find sampling rate F_s such that signal $x(n)$ reaches its peak value of 3? What is the minmum F_s suitable for this task?

Solution

(a) $1\ ms = 10^{-3}$ seconds

Calculate few values.

$t=0 \Rightarrow X_a(t) = 0$

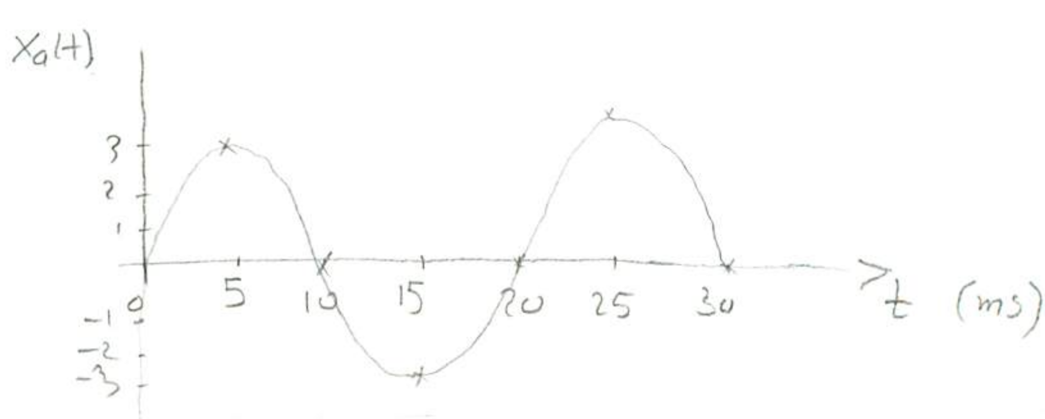
$t = 5ms \Rightarrow X_a(t) = 3\sin(100\pi\ 5\times10^{-3}) = 3\sin(.5\pi) = 3$

$t = 10ms \Rightarrow X_a(t) = 3\sin(100\pi\ 10\times10^{-3}) = 3\sin(\pi) = 0$

$t = 15ms \Rightarrow X_a(t) = 3\sin(100\pi\ 15\times10^{-3}) = 3\sin(1.5\pi) = -3$

$t = 20ms \Rightarrow X_a(t) = 3\sin(100\pi\ 20\times10^{-3}) = 0$

⋮



(b) $x_a(t) = A \sin(2\pi F t + \theta)$ —— ⓪

so $x_a(nT) = A \sin(2\pi F nT + \theta) = A \sin(2\pi \frac{F}{F_s} n + \theta)$ —— ①

but $x(n) = A \sin(2\pi f n + \theta)$ —— ②

hence by comparing ①, ② $\Longrightarrow$ $\boxed{f = \frac{F}{F_s}}$ $F \rightarrow$ cycles/sec, $F_s \rightarrow$ samples/sec.

now Find F and F_s to find f.

since $x_a(t) = 3 \sin(100\pi t)$

Then by comparing to ⓪ $\Longrightarrow$ $2\pi F t = 100\pi t \Longrightarrow \boxed{F = 50 \text{ cycles/sec}}$

$F_s = 300$ samples/sec.

hence $f = \frac{50}{300} = \boxed{\frac{1}{6} \text{ cycles/sample.}}$ ✓

since f is a rational number $\Longrightarrow$ $\boxed{\text{periodic}}$

and Fundemental period is $\boxed{6 \text{ samples}}$

(c) $x(n) = A \sin(2\pi f n + \theta)$. but $A = 3$, $\theta = 0$, $f = \frac{1}{6}$.

so $x(n) = 3 \sin(2\pi \frac{1}{6} n) = \boxed{3 \sin(\frac{\pi}{3} n)}$

$n = 1 \Rightarrow x(1) = 3 \sin(\frac{\pi}{3}) = 2.598$

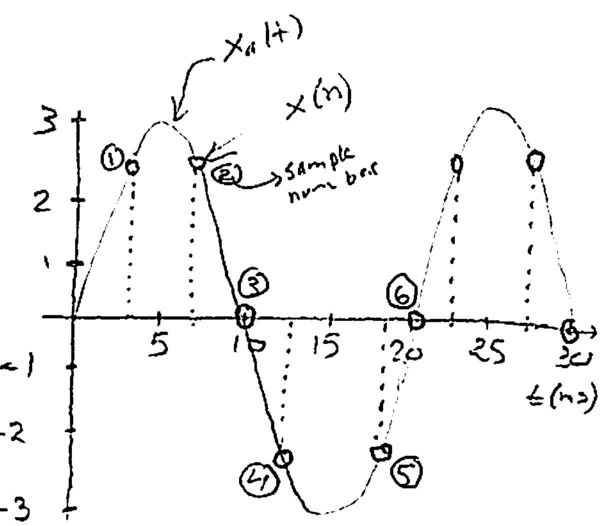
$n = 2 \Rightarrow x(2) = 3 \sin(\frac{2\pi}{3}) = 2.598$

$n = 3 \Rightarrow x(3) = 3 \sin(\pi) = 0$

$n = 4 \Rightarrow x(4) = 3 \sin(\frac{4}{3}\pi) = -2.598$

$n = 5 \Rightarrow x(5) = 3 \sin(\frac{5}{3}\pi) = -2.598$

$n = 6 \Rightarrow x(6) = 3 \sin(2\pi) = 0$



✓

since period of $x(n) = 6$ samples.

Then it takes $6 \times T$ seconds

or $6 \times \frac{1}{F_s} = 6 \frac{1}{300} = 0.02$ seconds $= \boxed{20 \text{ ms}}$ For period of $x(n)$ in ms.

(d)

Since $x_a(t)$ has a peak of 3 at $t = 5$ ms.

then need to solve

$$3 \sin(2\pi F t) = 3 \sin\left(2\pi \frac{F}{F_s} n\right)$$

let $t = 5 \times 10^{-3}$, and given $F = 50$ cycles/seconds. (From part b)

so $\sin\left(2\pi (50) 5\times10^{-3}\right) = \sin\left(2\pi \frac{50}{F_s} n\right)$

So

$$\boxed{1 = \sin\left(2\pi \frac{50}{F_s} n\right)}$$

so depending on n, we solve for F_s.

$$2\pi \frac{50}{F_s} n = \text{Arcsin}(1)$$

$$2\pi \frac{50}{F_s} n = m\frac{\pi}{2} \qquad \text{For } m = 1, 5, 9, 13, \ldots$$

4 So $\boxed{F_s = 200 \frac{n}{m}}$

choose $m=1$ $\Rightarrow$ $\boxed{F_s = 200\,n}$

The minimum is when $n=1$ $\Longrightarrow$ $\boxed{F_s = 200 \text{ samples/second.}}$

at This sampling rate, $x(n) = 3$ at sample number 1 after $T=5$ ms time. (sample period).