

EECS152A Exam #1 October 26, 2004

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I.D.:

This is an 80 minute, CLOSED BOOK exam. If you have any questions, please ask. GOOD LUCK!

Question 1: 10

Question 2: 20

Question 3: 15

Question 4: 15

Question 5: 10

Question 6: 7

Question 7: 17

TOTAL:

(94)

Question 1 (10 points) Consider the continuous-time signal

$$2\pi F_1 t \equiv 80\pi t \\ \Rightarrow F_1 = 40 \text{ Hz}$$

$$x(t) = 5 \sin 80\pi t + 3 \cos 25\pi t - \cos 70\pi t$$

a) What is the Nyquist rate for this signal?

$$F_1 = 40 \text{ Hz}, \quad F_2 = 12.5 \text{ Hz}, \quad F_3 = 35 \text{ Hz}.$$

so $F_{\max} = 40 \text{ Hz}$ \Rightarrow Nyquist freq = $2F_{\max}$

b) For what values of the sampling rate F_s will sinc interpolation allow us to recover $x(t)$ exactly? (3)

for values of $F_s >$ Nyquist frequency

✓ (3)

Question 2 (20 points) Consider the continuous-time signal

$$F = 25 \text{ Hz}$$

$$x_1(t) = 10 \cos(50\pi t + \pi)$$

Suppose that we sample $x_1(t)$ at a rate $F_s = 20 \text{ Hz}$ to generate the sampled signal $x_1(n)$.

a) Determine the sampled signal $x_1(n)$.

$$\begin{aligned} x_1(n) &= 10 \cos(50\pi(nT) + \pi) \\ &= 10 \cos(50\pi(\frac{n}{20}) + \pi) \\ x_1(n) &= 10 \cos(\frac{5}{2}\pi n + \pi) \\ \text{but } f &= \frac{5}{4} > |\frac{1}{2}| \Rightarrow f_s = \frac{1}{4} \Rightarrow x_1(n) = 10 \cos(2\pi(\frac{1}{4})n + \pi) \end{aligned}$$

b) If we apply sinc interpolation to $x_1(n)$, determine the recovered continuous-time signal $x'_1(t)$.

$$2\pi f_s n = \frac{5}{2}\pi n \Rightarrow f = \frac{5}{4} > |\frac{1}{2}| \text{ so aliasing.}$$

hence $f \rightarrow \frac{1}{4}$

$$\text{so } x_1(n) = 10 \cos(2\pi(\frac{1}{4})n + \pi).$$

$$\begin{aligned} \text{so } x'_1(t) &= 10 \cos(2\pi(\frac{1}{4})t + \pi) \\ &= 10 \cos(\frac{40}{4}\pi t + \pi) = 10 \cos(10\pi t + \pi) \end{aligned}$$

now, note that $T = \frac{1}{20} \text{ sec}$

c) Consider the continuous-time signal

✓ (8)

$$x_2(t) = 10 \cos(2\pi F t + \pi)$$

Suppose that we sample $x_2(t)$ at $F_s = 20 \text{ Hz}$ to generate the sampled signal $x_2(n)$. Determine all values of F for which the signals $x_2(n)$ and $x_1(n)$ are equal for all samples n .

$$\begin{aligned} x_2(n) &= 10 \cos(2\pi F(nT) + \pi) \\ &= 10 \cos(2\pi \frac{F}{F_s} n + \pi) \end{aligned}$$

$$x_2(n) = 10 \cos(2\pi \frac{F}{20} n + \pi) \quad \text{but } x_1(n) = 10 \cos(2\pi \frac{1}{4} n + \pi).$$

$$\text{So solve } 2\pi \frac{F}{20} n = 2\pi \frac{1}{4} n$$

(7)

$$\text{so } F = 5 \text{ Hz}$$

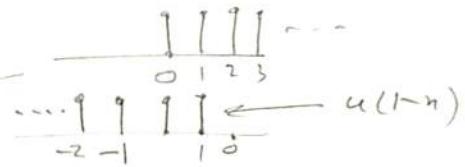
$$\text{so } F = F_0 + kF_s$$

$k = \pm 1, \pm 2, \dots$

$$F = 5, 25, 45, 65, \dots, -15, -35, -55, \dots$$

Question 3 (15 points) a) Compute the z-transform of

$$x(n) = \left(\frac{3}{2}\right)^n u(1-n)$$



$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{\infty} \left(\frac{3}{2}\right)^n u(1-n) z^{-n}$$

$$X(z) = \sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n z^{-n}$$

$$= \sum_{-1}^{\infty} \left(\frac{3}{2}\right)^{-n} z^n = \left[\sum_{0}^{\infty} \left(\frac{3}{2}\right)^{-n} z^n \right] + \left(\frac{3}{2} \frac{1}{z}\right)$$

$$= \sum_{0}^{\infty} \left(\frac{2z}{3}\right)^n + \left(\frac{3}{2} \frac{1}{z}\right) = \frac{1}{1 - \frac{2z}{3}} + \frac{3}{2} \frac{1}{z}$$

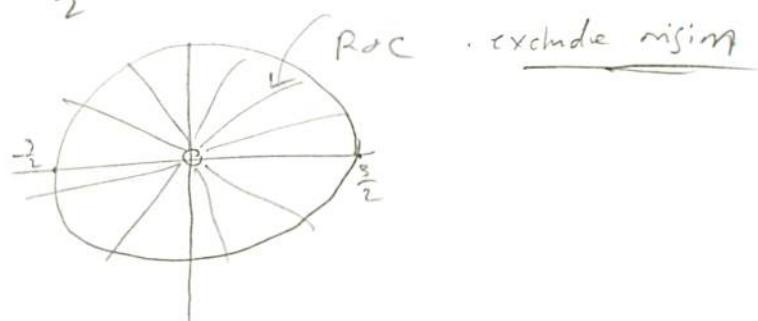
$$= \frac{2z + 3(1 - \frac{2z}{3})}{2z - \frac{4}{3}z^2} = \frac{2z + 3 - 2z}{2z(1 - \frac{1}{3}z)} = \boxed{\frac{3}{2z(1 - \frac{1}{3}z)}}$$

✓ 10

b) What is the region of convergence?

$$\left| \frac{2}{3}z \right| < 1 \quad \text{or} \quad |z| < \frac{3}{2}$$

✓ 5



Question 4 (15 points) Consider the system

$$y(n) = 2x(n) + 4$$

a) Is this system linear?

NO. ✓ (5)

$$\begin{aligned} T[a x_1(n) + b x_2(n)] &\stackrel{?}{=} a T[x_1(n)] + b T[x_2(n)] \\ 2[a x_1(n) + b x_2(n)] + 4 &\stackrel{?}{=} a[2 x_1(n) + 4] + b[2 x_2(n) + 4] \\ 2a x_1(n) + 2b x_2(n) + 4 &\stackrel{?}{=} 2ax_1(n) + \underline{4a} + 2bx_2(n) + \underline{4b} \end{aligned}$$

b) Write down the definition of a linear system. Use the definition of linear system to justify your answer to part a).

definition is: if $T[a x_1(n) + b x_2(n)] = a T[x_1(n)] + b T[x_2(n)]$

for any a, b , and for any $x_1(n), x_2(n)$ (5)

by applying this definition, we see that $LHS \neq RHS$

for all a, b . it is only true for $a = \frac{1}{2}, b = \frac{1}{2}$, but not for say $a = 1, b = 1$.

Hence NOT Linear (5)

Question 5 (10 points) Consider the system

$$y(n) = \sum_{k=0}^n x(k) = x(0) + x(1) + x(2) + \dots + x(n)$$

a) Is this system time-invariant?

NO

✓. (5)

b) Justify your answer to part a).

A system is time invariant if delayed input produces the delayed output of the input when the input was not delayed.

i.e. $y(n, L) = y(n-L)$ for all delay L .

$y(n, L) = \sum_{k=0}^n x(k-L)$	$= x(-L) + x(1-L) + x(2-L) + \dots + x(n-L)$
$y(n-L) = \sum_{k=0}^{n-L} x(k)$	$= x(0) + x(1) + x(2) + \dots + x(n-L)$

we see $y(n, L) = y(n-L)$ only for $L=0$.

\Rightarrow NOT time-invariant

for example, for $n=3$, $L=1$ we set

$$y(n, L) = x(-1) + x(0) + x(1) + x(2)$$

$$y(n-L) = x(0) + x(1) + x(2)$$

(5)

Question 6 (10 points) Let $x(n) = \delta(n)$ and let $y(n) = \left(\frac{1}{2}\right)^n u(n)$. Compute the crosscorrelation $r_{xy}(l)$ of $x(n)$ and $y(n)$.

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n) y(n-l)$$

$$r_{xy}(l) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \delta(n-l)$$

but $\delta(n-l) = 0$ for all $n-l \neq 0$
 i.e. when $n \neq l \Rightarrow \delta(n-l) = 1$
 & when $l=n \Rightarrow \left(\frac{1}{2}\right)^l$

so
$$r_{xy}(l) = \left(\frac{1}{2}\right)^l$$

①

Question 7 (20 points) The following input-output pairs are observed during the operation of a linear time-invariant system

$$x_1(n) = \{0, 0, 4\} \longleftrightarrow y_1(n) = \{0, 4, 6, -8\}$$

↑ ↑

$$x_2(n) = \{2, 4\} \longleftrightarrow y_2(n) = \{2, 7, 2, -8\}$$

↑ ↑

a) From this information, is it possible to determine the output $y_3(n)$ of the system for the input

YES

$$x_3(0) = 1 \quad x_3(1) = -1$$

Assume that $x_3(n)$ is zero for all other values of n .

note: if input is delta, then output is impulse response $h(n)$

b) If you answered no to part a), explain why not. If you answered yes to part a), determine the output sequence $y_3(n)$. I only need to use $x_1(n), y_1(n)$.

Find $H(z)$

$$\text{from } \boxed{x_1(n), y_1(n)} \Rightarrow X_1(z) = 4z^{-2}$$

$$Y_1(z) = 4z^{-1} + 6z^{-2} - 8z^{-3}$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{4z^1 + 6z^{-2} - 8z^{-3}}{4z^{-2}} = \boxed{z + \frac{3}{2} - 2z^{-1}}$$

$$\therefore \boxed{h(n) = \left\{ 1, \frac{3}{2}, -2, 0, \dots \right\}} \quad \checkmark \quad \text{now can use } H(z) \text{ to find}$$

$y_3(n)$ for other inputs:

$$Y_3(z) = H(z)X_3(z) = \left(z + \frac{3}{2} - 2z^{-1} \right) \boxed{(1)} \Rightarrow \boxed{y_3(n) = \left\{ 1, \frac{3}{2}, -2 \right\}}$$

for $x_3(1) = -1$, we set

$$Y_3(z) = H(z)X_3(z) = \left(z + \frac{3}{2} - 2z^{-1} \right) (-1) \Rightarrow \boxed{y_3(n) = \left\{ -1, -\frac{3}{2}, 2 \right\}}$$

↑