

**Instruction:**

1. **Print this question paper and show the necessary steps on the space provided under each question.**
2. **If you don't have access to printer, use white paper.**
3. **Use black ink and write bigger fonts so that it is visible when posted and printed again.**
4. **Follow the instruction** given for each question.
5. **You can only post once. So make sure all the pages/questions are posted properly.**
6. **Post on D2L as one PDF file by July 17, 2021.**
7. **Write your name on the answer sheet.**

**MATH2520    CALCULUS 4    FINAL EXAM**

**Name:** \_\_\_\_\_

**INSTRUCTION:** Show all the necessary steps on the space provided under each question or on a separate sheet.

1. Solve the following system of equations and write the solution as a parametric vector form. **( 4 pts )**

$$x + 2y - 3z = 5$$

$$2x + y - 3z = 13$$

$$-x + y = -8$$

2. Compute the determinant using a cofactor expansion. **( 3 pts )**

$$\begin{vmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{vmatrix}$$

3. Let  $A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$  and  $\mathbf{u} = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$ .

a) Is  $\mathbf{u}$  in Nullspace( A )? Justify your answer.

( 3 pts )

b) Is  $\mathbf{u}$  in Columnspace( A )? Justify your answer.

( 3 pts )

c) Determine the rank A and Nullity of A. Show your work.

( 2 pts )

4. a) Using the definition, verify that the given transformation is linear transformation.  
 $T: C^2(I) \rightarrow C^0(I)$  defined by  $T(y) = y'' + y$  (4 pts)

- b) Find the kernel of  $T$ . (4 pts)

5. Solve: ( 5 pts )  
 $(y + 3x^2)dx + xdy = 0$

6. Using the **method of undermined coefficients**, find the general solution of the given differential equation. ( 10 pts )

$$y'' - y' - 2y = e^{-x} + 2 \cos x$$

7. Use the **Laplace transform** to solve the given initial-value problems. ( 10 pts )  
You can use the table of transformation.

$$y'' + y = e^{2t}, \quad y(0) = 0, \quad y'(0) = 1$$

8. Find a **series solution** in powers of  $x$  of the differential equation. ( 10 pts )

$$y'' + x^2 y' + y = 0$$



9. a) Determine all the equilibrium points of the given system.  
b) Select two equilibrium points and classify them as saddle, node, spiral or center and whether they are stable or unstable.

**( 10 pts)**

$$x' = 2x - x^2 - xy$$

$$y' = 3y - 3xy - 2y^2$$