Math2520 Calculus IV Mid-Semester Exam.

Name: _____

INSTRUCTION:

1. Write you name on the answer sheet.
2. Write clearly and legibly (bigger and darker) so that it easy to read when printed.
3. You can only post once, so make sure that all the pages/questions are posted.
4. You can use your own paper if you cannot print it.

1. Solve the initial-value problem. (**4 pts**)

$$x\frac{dy}{dx} - y = 2x^2 y, \quad y(1) = 1$$

2. Solve: (**5 pts**)

$$x^2 \frac{dy}{dx} + 2xy - y^3 = 0, x > 0$$

3. Verify that the given differential equation is exact; then solve it. (**6 pts**)

$$(x^3 + \frac{y}{x})dx + (y^2 + \ln x)dy = 0$$

4. a) Solve the initial value problem (**4 pts**)

$$\frac{dy}{dx} = 3 + x - y, \quad y(0) = 1$$

b) Apply Euler's methods to the initial value problem with step size h = 0.1 and complete the following table. You can use calculator or excel. (**4 pts**)

x	Euler method y	Exact y	Absolute Error
0.1			
0.2			
0.3			
0.4			

5. Solve the following system of equations and write the solution in parametric vector form.

(**4 pts**)

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ 2x_1 - x_2 + 2x_3 &= 2 \\ 3x_1 + x_2 + 3x_3 &= -8 \end{aligned}$$

6. Given the matrix $A = \begin{bmatrix} 3 & 4 \\ 4 & -2 \end{bmatrix}$, **(5 pts)**

a) Find A^{-1} , the inverse matrix of *A*.

b) Use A^{-1} to solve the system of equations

$$3x + 4y = 7$$
$$4x - 2y = 5$$

7. Use the cofactor expansion to evaluate the given determinant along the 2nd row.

$$\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{vmatrix}$$

8. Let H be the set of points in the $xy-plane$ given by,

$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$. Show that H is not a subspace of R^2. (**3 pts**)

9. Determine if the set of vectors span R^3. Justify our answer. (**3 pts**)

$\{(1,-2,1),(2,3,1),(4,-1,2)\}$

10. Mark each statement **TRUE** or **FALSE**. (**5 pts**)

a) An integrating factor for the differential equation $\frac{dy}{dx} = x^2 y$ is $e^{\int x^2 dx}$. _____

b) The equation $Ax = 0$ has the nontrivial solution if and only if there are free variables._____

c) If A is $n \times n$ matrix, then $\det(cA) = c \det A$, c constant. _____

d) The solution set of a homogeneous linear system $Ax = 0$ of *m* equation and *n* unknowns is a subspace of R^n. _____

e) If **x** is a vector in the first quadrant of R^2, then any scalar multiple *k***x** of **x** is still a vector in the first quadrant of R^2. _____