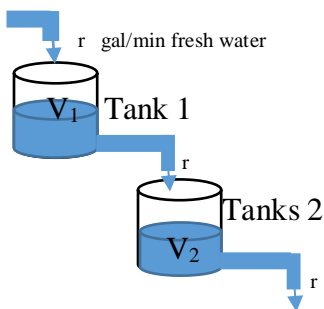


INSTRUCTION: *Show all the necessary work.* Write your answer on a separate sheet preferably hand written clear and legible. Post your answer sheet on D2L by **Sunday July 11.**

1. Consider the following figure below that shows two brine tanks



containing V_1 and V_2 gallons of brine respectively. Fresh water flows into tank 1, while mixed brine flows from tank 1 into tank 2. Let $x_i(t)$ denote the amount (in pounds) of salt in tank i at time t for $i=1,2$. If each flow rate is r gallons per minute, then a simple account of salt concentration yields the first-order system

$$\begin{aligned}x_1' &= -k_1 x_1 \\x_2' &= k_1 x_1 - k_2 x_2\end{aligned}$$

where

$$k_i = \frac{r}{V_i}, \quad i=1,2.$$

If $V_1 = 25$, $V_2 = 50$, $r = 10$ (gal / min), and the initial amounts of salt in the two brine tanks, in pounds, are

$$x_1(0) = 15, \quad x_2(0) = 0,$$

- Find the amount of salt in each tank at time $t \geq 0$.
- Find the maximum amount of salt ever in tank 2.

2. Determine all the equilibrium points of the given system.

$$x' = x - x^2 - xy$$

$$y' = 3y - xy - 2y^2$$

3. Using the definition of Laplace transform, determine $L\{f\}$.

$$f(t) = te^t$$

4. Find the inverse Laplace transform of the given functions.

a) $F(s) = \frac{2}{s(s-2)}$

b) $F(s) = \frac{2s+2}{s^2+2s+5}$

5. Use the Laplace transform to solve the following given initial-value problems.

a) $y' + y = 8e^{3t}$, $y(0) = 2$

b) $y'' + y' - 2y = 10e^{-t}$, $y(0) = 0$, $y'(0) = 1$