

11.6-3

$$[\underline{K}] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}, \quad [\underline{M}] = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$
$$K_{ss} = k, K_{ms} = -k, [\underline{T}] = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad [\underline{T}]^T [\underline{K}] [\underline{T}] = 0$$
$$[0 - \omega^2(2m)] \bar{u}_i = 0 \quad [\underline{T}]^T [\underline{M}] [\underline{T}] = 2m$$

Hence  $\omega = 0$ ; the rigid body mode; OK.

11.6-4

(a) With  $c$  a constant,  $\frac{M_{11}}{K_{11}} = c \frac{156}{12}$

and  $\frac{M_{22}}{K_{22}} = c \frac{4L^2}{4L^2} = c$ .  $\frac{M_{11}}{K_{11}} > \frac{M_{22}}{K_{22}}$ , therefore

the choice is proper.

(b)  $[\tilde{T}] = \begin{bmatrix} -\frac{L^3}{12EI} & \frac{6EI}{L^2} \\ 1 & 1 \end{bmatrix} = \begin{Bmatrix} -\frac{L}{2} \\ 1 \end{Bmatrix}$

$$[\tilde{T}]^T [\tilde{K}] [\tilde{T}] = \left[ -\frac{L}{2} \ 1 \right] \frac{EI}{L^3} \begin{Bmatrix} -6L + 6L \\ -3L^2 + 4L^2 \end{Bmatrix} = \frac{EI}{L}$$

$$[\tilde{T}]^T [\tilde{M}] [\tilde{T}] = \left[ -\frac{L}{2} \ 1 \right] \frac{m}{420} \begin{Bmatrix} -78L - 13L \\ 6.5L^2 + 4L^2 \end{Bmatrix} = \frac{14mL^2}{105}$$

$$\left( \frac{EI}{L} - \omega^2 \frac{14mL^2}{105} \right) \bar{\theta}_2 = 0, \quad \omega^2 = 7.5 \frac{EI}{mL^3}$$

11.7-2

Let  $\bar{D}_i^*$  be a vector before normalization.

Evaluate  $c$  in  $(\bar{D}_i^*)^T M \bar{D}_i^* = c$

Scaled vector  $\bar{D}_i = \frac{1}{\sqrt{c}} \bar{D}_i^*$  will yield  $\bar{D}_i^T M \bar{D}_i = 1$

From Prob. 11.4-2a,  $\omega_1^2 = 1$ ,  $\omega_2^2 = 6$ , and  $c = \sqrt{5}$

Now use Eqs. 11.7-5 and 11.7-6

$$[\phi] = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$$

$$\begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \frac{1}{\sqrt{5}} \begin{Bmatrix} 2R_1 + R_2 \\ R_1 - 2R_2 \end{Bmatrix}$$

Eq. 13.6-5 yields

$$\ddot{z}_1 + z_1 = (2R_1 + R_2)/\sqrt{5}$$

$$\ddot{z}_2 + 6z_2 = (R_1 - 2R_2)/\sqrt{5}$$

11.7-4

(a)

$$\left( \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} - \omega^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{Bmatrix} \bar{u}_1 \\ \bar{u}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Solution of this eigen problem yields

$$\lambda_1 = 0.381966, \quad \omega_1 = 0.618034$$

$$\lambda_2 = 2.61803, \quad \omega_2 = 1.618034$$

Eigenvectors, respectively un-normalized and normalized, are

$$\begin{Bmatrix} 1 \\ 1.61803 \end{Bmatrix} \rightarrow \begin{Bmatrix} 0.52573 \\ 0.85065 \end{Bmatrix}, \quad \begin{Bmatrix} 1 \\ -0.618034 \end{Bmatrix} \rightarrow \begin{Bmatrix} 0.85065 \\ -0.52573 \end{Bmatrix}$$

Hence  $\begin{Bmatrix} \dot{\phi} \\ \ddot{\phi} \end{Bmatrix} = \begin{bmatrix} 0.52573 & 0.85065 \\ 0.85065 & -0.52573 \end{bmatrix}$

Initial conditions, using Eq. 11.7-4, are

$$\begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \quad \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} = [\dot{\phi}]^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0.85065 \\ -0.52573 \end{Bmatrix}$$

With  $\xi_i = p_i = 0$ , Eq. 11.7-6 has the solution

$$z_i = A_i \sin \omega_i t + B_i \cos \omega_i t$$

$B_i = 0$  because  $z_i = 0$  at  $t = 0$ .

Next,  $\dot{z}_i = A_i \omega_i \cos \omega_i t$ , and at  $t = 0$

$$\dot{z}_1 = 0.85065 = A_1 (0.618) \quad \left. \right\} A_1 = 1.3764$$

$$\dot{z}_2 = -0.52573 = A_2 (1.618) \quad \left. \right\} A_2 = -0.3249$$

$$z_1 = 1.3764 \sin 0.618t$$

$$z_2 = -0.3249 \sin 1.618t$$

By Eq. 11.7-4,  $\{D\} = [\dot{\phi}]^T \{\ddot{z}\}$  yields

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0.7236 \sin 0.618t - 0.2764 \sin 1.618t \\ 1.171 \sin 0.618t + 0.1708 \sin 1.618t \end{Bmatrix}$$

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
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$$u_1 \quad 0 \quad .1432 \quad .7095 \quad .9684 \quad .3972 \quad -.2315$$

$$u_2 \quad 0 \quad .8491 \quad 1.090 \quad .9552 \quad .7589 \quad .2262$$

(c) Compare greatest magnitudes, regardless of the times at which they appear.

$$\frac{0.7236}{0.7236 + 0.2764} = 0.7236 \quad (27.6\% \text{ error in } u_1)$$

$$\frac{1.171}{1.171 + 0.1708} = 0.8727 \quad (12.7\% \text{ error in } u_2)$$