

3.6-3 Beam theory will give  $u_F = -u_D$ ,  $v_F = v_D$

(a) From Eq. 3.6-6,

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{4ab} \begin{bmatrix} \dots & b-y & 0 & b+y & 0 & \dots \\ \dots & 0 & -(a+x) & 0 & a+x & \dots \\ \dots & -(a+x) & b-y & a+x & b+y & \dots \end{bmatrix} \begin{Bmatrix} \vdots \\ u_D \\ v_D \\ -u_D \\ v_D \\ \vdots \end{Bmatrix}$$

From beam theory, Prob. 3.4-1,  $u_D = -\frac{3ML}{2Etc^2}$ ,  $v_D = -\frac{3ML^2}{4Etc^3}$

Hence

$$\epsilon_x = \frac{3MLy}{4Etabc^2}, \quad \epsilon_y = 0, \quad \gamma_{xy} = \frac{3ML(a+x)}{4abEtc^2} - \frac{3ML^2}{8aEtc^3}$$

$$\text{For } v=0, \quad \sigma_x = E\epsilon_x = \frac{3MLy}{4tabc^2}, \quad \sigma_y = E\epsilon_y = 0,$$

$$\tau_{xy} = \frac{E}{2} \gamma_{xy} = \frac{3ML(a+x)}{8abtc^2} - \frac{3ML^2}{16atc^3}$$

$$\text{But } L=2a \text{ and } c=b, \text{ so } \sigma_x = \frac{3My}{2tb^3}, \quad \sigma_y = 0, \quad \tau_{xy} = \frac{3Mx}{4b^3t}$$

(in the coordinate system of Fig. 3.6-1). In this system,  
 $\sigma_x = \frac{My}{I} = \frac{3My}{2tb^3}$ ,  $\sigma_y = \tau_{xy} = 0$  according to beam theory.

3.6-5

Method 1: Evaluate  $u = a_1 + a_2x + a_3y + a_4xy$  at nodes, solve for  $a$ 's, gather coefficients of  $u_1, u_2, u_3, u_4$ .

$$u_1 = a_1$$

Node 1

$$u_2 = u_1 + a_2(2a); a_2 = \frac{u_2 - u_1}{2a} \quad \text{Node 2}$$

$$u_4 = u_1 + a_3(2b); a_3 = \frac{u_4 - u_1}{2b} \quad \text{Node 4}$$

$$u_3 = u_1 + \frac{u_2 - u_1}{2a} 2a + \frac{u_4 - u_1}{2b} 2b + a_4(2a)(2b)$$

$$\text{from which } a_4 = \frac{u_1 - u_2 + u_3 - u_4}{4ab}$$

$$u = u_1 + \frac{u_2 - u_1}{2a} x + \frac{u_4 - u_1}{2b} y + \frac{u_1 - u_2 + u_3 - u_4}{4ab} xy$$

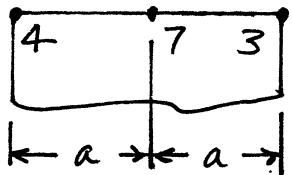
$$u = \left(1 - \frac{x}{2a} - \frac{y}{2b} + \frac{xy}{4ab}\right)u_1 + \left(\frac{x}{2a} - \frac{xy}{4ab}\right)u_2$$

$$+ \left(\frac{xy}{4ab}\right)u_3 + \left(\frac{y}{2b} - \frac{xy}{4ab}\right)u_4$$

Coefficients of the  $u_i$  are the  $N_i$ .

3.11-1

$$\text{Eq. 3.11-6 : } \begin{Bmatrix} F_4 \\ F_7 \\ F_3 \end{Bmatrix} = \frac{a}{15} \underbrace{\begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix}}_{[H]} \begin{Bmatrix} q_4 \\ q_7 \\ q_3 \end{Bmatrix}$$



Unit thickness

$$(a) \begin{Bmatrix} F_4 \\ F_7 \\ F_3 \end{Bmatrix} = [H] \begin{Bmatrix} \sigma \\ 0 \\ -\sigma \end{Bmatrix} = \frac{\sigma a}{3} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$$

Couple-moment:  $M = F_4(2a) = \frac{2\sigma a^2}{3}$   
Flexure formula:  
 $M = \frac{\sigma I}{c} = \frac{\sigma (2a)^3/12}{a} = \frac{2\sigma a^2}{3}$

$$(b) \begin{Bmatrix} F_4 \\ F_7 \\ F_3 \end{Bmatrix} = [H] \begin{Bmatrix} 0 \\ \sigma/2 \\ \sigma \end{Bmatrix} = \frac{\sigma a}{3} \begin{Bmatrix} 0 \\ 2 \\ 1 \end{Bmatrix}$$

Moment:  $M = 2aF_3 + aF_7 = \frac{4\sigma a^2}{3}$   
Flexure formula, for section 4a units deep:  
 $M = \frac{\sigma I}{c} = \frac{\sigma (4a)^3/12}{2a} = \frac{8\sigma a^2}{3}$

Contribution of half the section is  
 $\frac{M}{2} = \frac{4\sigma a^2}{3}$

$$(c) \begin{Bmatrix} F_4 \\ F_7 \\ F_3 \end{Bmatrix} = [H] \begin{Bmatrix} 0 \\ T \\ 0 \end{Bmatrix} = \frac{T a}{15} \begin{Bmatrix} 2 \\ 16 \\ 2 \end{Bmatrix}$$

Shear force:  $F_4 + F_7 + F_3 = \frac{4Ta}{3}$   
Beam theory (parabolic distribution):  
 $T = \frac{3}{2} \frac{V}{2a}, V = \frac{4Ta}{3}$

3.11-2

With  $\underline{[N]}$  from Eq. 3.11-5, and constant  $q$ ,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \int_{-a}^a \underline{[N]} q dx = \int_{-a}^a \frac{1}{2a^2} \begin{Bmatrix} x(x-a) \\ 2(a^2-x^2) \\ x(x+a) \end{Bmatrix} q dx$$

$$= \frac{q}{2a^2} \left\{ \begin{Bmatrix} \frac{x^3}{3} - \frac{ax^2}{2} \\ 2a^2x - \frac{2x^3}{3} \\ \frac{x^3}{3} + \frac{ax^2}{2} \end{Bmatrix} \right\}_{-a}^a = \frac{q}{2a^2} \left\{ \begin{Bmatrix} 2a^3/3 \\ 8a^3/3 \\ 2a^3/3 \end{Bmatrix} \right\} = q \left\{ \begin{Bmatrix} a/3 \\ 4a/3 \\ a/3 \end{Bmatrix} \right\} = F \left\{ \begin{Bmatrix} 1/6 \\ 2/3 \\ 1/6 \end{Bmatrix} \right\}$$

where  $F = 2qa$