$\boxed{3.3\text{-}1}$

(a) $u = \lfloor 1 \quad x \rfloor \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}, \quad \lfloor \underset{\sim}{B}_a \rfloor = \dfrac{\partial}{\partial x} \lfloor 1 \quad x \rfloor = \lfloor 0 \quad 1 \rfloor$

$$[\underset{\sim}{k}_a] = \int_0^L \lfloor \underset{\sim}{B}_a \rfloor^T \lfloor \underset{\sim}{B}_a \rfloor AE\, dx = AEL \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Let $x_1 = 0$, $x_2 = L$ in Eq. 3.2-4. Then $[\underset{\sim}{A}] = \begin{bmatrix} 1 & 0 \\ 1 & L \end{bmatrix}$, $[\underset{\sim}{A}]^{-1} = \begin{bmatrix} 1 & 0 \\ -1/L & 1/L \end{bmatrix}$

$$[\underset{\sim}{k}] = [\underset{\sim}{A}]^{-T} [\underset{\sim}{k}_a] [\underset{\sim}{A}]^{-1} = \begin{bmatrix} 1 & -1/L \\ 0 & 1/L \end{bmatrix} \left(AEL \begin{bmatrix} 0 & 0 \\ -1/L & 1/L \end{bmatrix} \right) = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \checkmark$$

(b) $v = \lfloor 1 \quad x \quad x^2 \quad x^3 \rfloor \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}, \quad \dfrac{d^2 v}{dx^2} = \lfloor 0 \quad 0 \quad 2 \quad 6x \rfloor \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} = \lfloor \underset{\sim}{B}_a \rfloor \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix}$

$$[\underset{\sim}{k}_a] = \int_0^L \lfloor \underset{\sim}{B}_a \rfloor^T \lfloor \underset{\sim}{B}_a \rfloor EI\, dx = EI \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4L & 6L^2 \\ 0 & 0 & 6L^2 & 12L^3 \end{bmatrix}$$

$[\underset{\sim}{A}]^{-1}$ is calculated in Problem 3.2-4

$$[\underset{\sim}{k}] = [\underset{\sim}{A}]^{-1} [\underset{\sim}{k}_a] [\underset{\sim}{A}]^{-1} = \begin{bmatrix} 1 & 0 & -3/L^2 & 2/L^3 \\ 0 & 1 & -2/L & 1/L^2 \\ 0 & 0 & 3/L^2 & -2/L^3 \\ 0 & 0 & -1/L & 1/L^2 \end{bmatrix} \left(EI \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \\ 6 & 0 & -6 & 6L \end{bmatrix} \right)$$

$$[\underset{\sim}{k}] = EI \begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ 6/L^2 & 4/L & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 12/L^3 & -6/L^2 \\ 6/L^2 & 2/L & -6/L^2 & 4/L \end{bmatrix} \checkmark$$

$\boxed{3.4\text{-}3}$

(a) The only nonzero d.o.f. are u_3 and v_3. From Eq. 3.4-10,

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = [\underset{\sim}{B}]\{\underset{\sim}{d}\} = \begin{bmatrix} 0 & 0 \\ 0 & 1/a \\ 1/a & 0 \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} . \quad \text{For } \nu = 0,$$

$$[\underset{\sim}{k}] = Et\frac{a^2}{2}\begin{bmatrix} 0 & 0 & 1/a \\ 0 & 1/a & 0 \end{bmatrix} \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$[\underset{\sim}{k}] = \frac{Et}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1/2 & 0 \end{bmatrix} = \frac{Et}{2}\begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$$