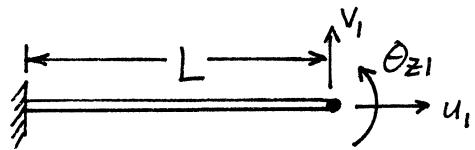


$$2.5-4 \quad a = AE/L \quad b = EI/L^3$$



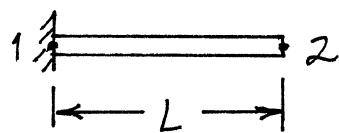
$$[\tilde{K}] = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 12b & -6bL & 0 \\ 0 & -6bL & 4bL^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} u_1 \\ v_1 \\ \theta_{z1} \\ \theta_{z2} \end{array}$$

$$[\tilde{K}] = \begin{bmatrix} 12b & 0 & 6bL & 6bL \\ 0 & a & 0 & 0 \\ 6bL & 0 & 4bL^2 & 2bL^2 \\ 6bL & 0 & 2bL^2 & 4bL^2 \end{bmatrix} \begin{array}{l} u_1 \\ v_1 \\ \theta_{z1} \\ \theta_{z2} \end{array}$$

Add; get

$$[\tilde{K}] = \begin{bmatrix} a+12b & 0 & 0 & 0 \\ 0 & a+12b & -6bL & 0 \\ 6bL & -6bL & 8bL^2 & 2bL^2 \\ 6bL & 0 & 2bL^2 & 4bL^2 \end{bmatrix}$$

2.7-1



(a)

From Eq. 2.3-5,

$$EI_z \begin{bmatrix} 12/L^3 & -6/L^2 \\ -6/L^2 & 4/L \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} R_2 \\ M_2 \end{Bmatrix} \quad (A)$$

Set $v_2 = \bar{v}_2$

$$EI_z \begin{bmatrix} 1 & 0 \\ 0 & 4/L \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} EI_z \bar{v}_2 \\ 6EI_z \bar{v}_2 / L^2 \end{Bmatrix} \text{ gives } \theta_{z2} = \frac{3\bar{v}_2}{2L}, v_2 = \bar{v}_2$$

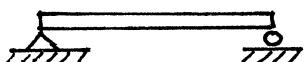
$$\text{Eq. (A) then gives } R_2 = EI_z \left[\frac{12}{L^3} \bar{v}_2 - \frac{6}{L^2} \frac{3\bar{v}_2}{2L} \right] = \frac{3EI_z \bar{v}_2}{L^3}$$

$$\text{Beam theory: } v_2 = \frac{R_2 L^3}{3EI_z}, \quad \theta_{z2} = \frac{R_2 L^2}{2EI_z}$$

$$\theta_{z2} = \underbrace{\frac{L^2}{2EI_z}}_{\frac{3EI_z}{L^3}} \bar{v}_2 = \frac{3\bar{v}_2}{2L}$$

$$R_2 = \frac{2EI_z}{L^2} \theta_{z2} = \frac{3EI_z}{L^3} v_2$$

(b)



From Eq. 2.3-5,

$$\frac{2EI_z}{L} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \theta_{z1} \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} \quad (B)$$

Set $\theta_{z1} = \bar{\theta}_{z1}$

$$\frac{2EI_z}{L} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \theta_{z1} \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} 2EI_z \bar{\theta}_{z1} / L \\ -2EI_z \bar{\theta}_{z1} / L \end{Bmatrix} \text{ gives } \theta_{z1} = \bar{\theta}_{z1}, \theta_{z2} = -\frac{\bar{\theta}_{z1}}{2}$$

$$\text{Eq. (B) then gives } M_1 = \frac{2EI_z}{L} \left[2\bar{\theta}_{z1} - \frac{\bar{\theta}_{z1}}{2} \right] = \frac{3EI_z}{L} \bar{\theta}_{z1}$$

These results appear in tables of beam deflections.