my written Lecture notes, EE 420 Digital Filters
California State University, Fullerton
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Contents

These are my overview of lecture notes for course introduction to digital filters that I took at California state university, Fullerton in spring 2010. And also other study notes.

1 lecture 1, Monday January 25, 2010

This lecture was general review of signals. Difference between continuous, discrete and digital signals is given. We obtain discrete signal from continuous by sampling. Digital signal is obtained from discrete by quantization.

Then review was given of unit step function, delta function. Some examples shown. important ones:

1. $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & else \end{cases}$
2. $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & else \end{cases}$
3. $x[n] = \sum_{k=-\infty}^{n} \delta(k) = \begin{cases} 1 & n \geq 0 \\ 0 & else \end{cases} = u(n)$
4. $x[n] = \sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 1 & n \geq 0 \\ 0 & else \end{cases} = u(n)$
5. $x[n] = \sum_{k=-\infty}^{\infty} \delta(n-k) = \left\{ \begin{array}{c} 1\, n \geq 0 \\ 1\, n < 0 \end{array} \right\} = 1$
6. $u[n] - u[n-1] = \delta[n]$

To find if a discrete function is periodic for a given frequency.

First method:

From $e^{j\omega_0 n} = e^{j\omega_0(n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}$, where N is the period. Hence we need to have $e^{j\omega_0 N} = 1$ for periodic, or $\omega_0 N = 2\pi k$ for some integer k. Therefore, the condition for periodicity is that $\omega_0 N = 2\pi k$ or $\frac{\omega_0}{2\pi} = \frac{k}{N}$

make sure k, N are relatively prime. If this is true, then N is the fundamental period. Notice that $\frac{\omega_0}{2\pi} = f$, the frequency is samples per second.

2 Example 1

given $\omega_0 = \frac{3\pi}{5}$ find if $\cos(\omega_0 n)$ is periodic. First note that $\cos(\omega_0 n) = \cos\left(2\pi f n\right)$, hence $f = \frac{\omega_0}{2\pi} = \frac{3\pi}{10\pi} = \frac{3}{10}$. Now $\cos(\omega_0(n+N)) = \cos\left(2\pi f(n+N)\right) = \cos\left(2\pi \frac{3}{10}(n+N)\right) = \cos\left(2\pi \frac{3}{10} n + 2\pi \frac{3}{10} N\right)$, Hence if we set $N = 10$, then $2\pi \frac{3}{10} N$ will be an integer an integer multiple of 2π

Hence $N = 10$ is the period and this is periodic.

Second method: Once we find that f is rational, we can stop and say it is periodic. To find the period, make f to be lowest relatively prime numbers. Hence the period is the denominator . So, in this example, $f = \frac{3}{10}$, we see it is periodic right away since f is rational. so period is 10. This second method is faster.

3 Example 2

First method

given $\omega_0 = 3$ find if $\cos(\omega_0 n)$ is periodic. We see that $f = \frac{3}{2\pi}$.

$$\cos(\omega_0(n+N)) = \cos\left(2\pi f(n+N)\right) = \cos\left(2\pi \frac{3}{2\pi}(n+N)\right) = \cos\left(2\pi \frac{3}{2\pi} n + 2\pi \frac{3}{2\pi} N\right)$$

$= \cos\left(2\pi \frac{3}{2\pi} n + 3N\right)$. We see that we can't find an integer N to make $3N$ be a multiple of 2π. Hence not periodic.

Second method:

Since $f = \frac{3}{2\pi}$ is not rational, we stop. Not periodic.

4 lecture 2, Wednesday January 27, 2010

Define energy of signal as $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

Showed that any signal $x(n)$ can be written as sum of weighted shifted Dirac delta functions, hence

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

Using the above, derived the convolution equation for linear time invariant system as follows. Let system be T, hence we have

$$\begin{aligned} y(n) &= T[x(n)] \\ &= T\left[\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)\right] \\ &= \sum_{k=-\infty}^{\infty} T[x(k)\,\delta(n-k)] \end{aligned}$$

Now, for linear system, we know that $T\left[af(n)\right] = aT\left[f(n)\right]$ for constant a, hence the above becomes

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,T[\delta(n-k)]$$

Now, let the response of the system for $\delta(n-k)$ be called $h(n,k)$, hence the above becomes

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n,k)$$

Now, assuming this is an LTI system, then the time when the impulse occurred would not change the response, hence the above becomes

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

And the above is the convolution equation, written as $y(n) = x(n) \circledast h(n)$

Next, went over definition of linear system. Linear system is one where if $x_1 \rightarrow y_1$ and $x_2 \rightarrow y_2$ then output of system when the input is $ax_1 + bx_2$ must be the same as $ay_1 + by_2$. If it is not the same, then the system is not linear. Here, I assumed a, b are constants, and I meant by x_i as the input and by y_i as the output.

Next, went over how to check if system is linear or not. see my study notes. Next went over definition of time invariant, which is: Output of a delayed input is the same as delayed output of the input. (delay amount is same ofcourse). Then went over how to check for time invariant. see my study notes.

Final went over linear convolution of 2 sequences and showed an example. Easy to do. Flip $h(n)$, the move the flipped $h(h)$ sequence and slide it over $x(n)$. Each time multiply corresponding values and adding.

That was the end of the second lecture.

5 lecture 3, Monday Feb1, 2010

Example of linear convolution given, solve analytically by finding the regions of interests. These will be partial overlapping (both left and right end as needed) and full overlap. Example was $x(n) = u(n) - u(n-5)$ and $h(h) = \alpha^n u(n)$ for $|\alpha| < 0$

Circular convolution question will be on final exam

More definitions given: Casual system: System can't predict input and react to it before it occurs. If system is LTI and casual, then $h(n) = 0$ for $n < 0$. Casual systems work in read time.

Definition of stability. BIBO. When checking for BIBO, remember to take the limit as $n \to \infty$ or $t \to \infty$ all the time. To check for BIBO, given a bounded input, say $|x(n)| \leq M$, and use this in the convolution to find $y(n)$ and see if $y(n)$ will be bounded as $n \to \infty$

Main theory: LTI system is stable iff

$$S = \sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Proof was given. Need to check for both directions here as this is an iff. Proof will not be on exam.

Examples given to check for stability: $y(n) = \sum_{k=0}^{n} x(k)$, to solve this, let $|x(n)| \leq M$, then

$$\begin{aligned} |y(n)| &= \left|\sum_{k=0}^{n} x(k)\right| \\ &\leq \sum_{k=0}^{n} |x(k)| \\ &= \sum_{k=0}^{n} M \\ &= (n+1) M \end{aligned}$$

Now, remember to take the limit $n \to \infty$, so we see that $y(\infty) \to \infty$ hence this is not stable system, since the input was bounded, but the output is not. Another example given is

$$y(n) = \sum_{k=0}^{n} a^k x(k)$$

Follow the same steps as above

$$\begin{aligned} |y(n)| &= \left|\sum_{k=0}^{n} a^k x(k)\right| \\ &\leq \sum_{k=0}^{n} \left|a^k x(k)\right| \\ &= \sum_{k=0}^{n} \left|a^k\right| |x(k)| \\ &\leq \sum_{k=0}^{n} \left|a^k\right| M \\ &= M \sum_{k=0}^{n} \left|a^k\right| \\ &= M \left(\frac{1-a^{n+1}}{1-a}\right) \end{aligned}$$

Hence, only for $|a| < 1$ will the above becomes $\frac{M}{1-a}$, ie. bounded. Hence system is table only for $|a| < 1$

Next, introduce difference equations as a way to describe LTI discrete system, an N order LTI system is

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_k y(n-r)$$

Examples: $y(n) = 3x(n)$, $y(n) = 3x(n) + 2x(n-3)$ and $y(n) + y(n-1) = x(n)$, this last one is different since y shows twice on the LHS. To solve this last one, let $x(n) = \delta(n)$ and find $h(n)$ (which will be $y(n)$ in this special case). Then go back and find $y(n)$ using convolution But remember, when letting $x(n) = \delta(n)$, we need to check for 2 cases, when $n = 0$ and when $n \neq 0$, and we have to assume values for $y(-1)$ and $y(0)$. We need additional condition to finally find $h(n)$.

Another way, is to solve the difference equation using Z transform. Which is what we will probably end up doing.

End of 3rd lecture.

6 Lecture 4, Wednesday, Feb. 3,2010

Given a difference equation, to solve for $y(n)$, do it in 2 steps: Let $x(n) = \delta(n)$, find find $y(n)$ which will be $h(n)$ in this case. Second step, find $y(n)$ for $x(n)$ using convolution.

Example given using $y(n) - ay(n) = x(n)$, solution shown assuming system is causal (i.e. $h(n) = 0$ for $n < 0$) and another solution shown for anti casual (i.e. $h(n) = 0$ for $n \geq 0$). Condition for stability is decided by $|a|$.

Mention that given a difference equation, without additional information, its solution is not unique.

Mention that if $h(n)$ has finite length, then system is called FIR, else it is called IIR

Then showed how to solve a difference equation using characteristic equation. Then showed circuit description of difference equation.

Introduction to Z transform, 2 sided. Definitions and examples. Then inverse Z transform. Idea is to write the Z transform as sums of $a_n z^{-n}$, then we can read the data directly since $x(n)$ will be the coefficient of the z^{-n} part. i.e. $x(-1)z^1 + x(0) + x(1)z^{-1} + x(2)z^{-2} + ...$, so using long division we can find $x(n)$

End of lecture 4.

7 Lecture 5, Monday, Feb. 8,2010

Example of finding inverse Z transform using partial fractions. Watch out if the sequence comes out unstable. rewrite for stable and not causal i.e. $u(-n-1)$ and do only for the part that is not stable., leave the other using $u(n)$

Now using one sided Z transform, the Z transform of a delayed sequence is generated. $Z(x[n-1]) = z^{-1}Z(x[n]) + x(-1)$

Next an example of a difference equation is given, with an initial condition, and the Z transform is used to solve it. Do not use $u(n)$ in the answer, since initial condition is at $n = -1$, just write $n \geq 0$

Next, we are given an LSI system with $e^{j\omega n}$ as input, and proofed that output $y(n) = x(n) H(j\omega)$ where $H(j\omega)$ is the frequency response. it is the DTFT of $h(n)$ of the system. i.e. $H(j\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$.

$H(j\omega)$ is periodic of period 2π and we normally use $\omega = -\pi \cdots \pi$ as the range. Note ω is continuous and has units of radians (I thought it was in units radians/sample).

Next, we learned how to express $H(j\omega)$ as $|H(j\omega)| e^{j\arg(H(j\omega))}$ to make it easier to draw the magnitude and phase diagrams of the frequency response of the system. The trick to use is to factor out $e^{ja\omega}$ out and to end up with expression as $A(\omega) e^{j\arg(H)}$. We are shown 2 sequences $x(n)$ and asked to find its *DTFT* and put the result in this form. Next, the magnitude and phase diagrams are plotted. Remember the following: Since we are using absolute value on the magnitude, when $\omega < 0$ and we get a negative value for the absolute, we multiply it by -1 to get +ve, then for that region of ω remember to add a π when doing the phase diagram.

HW1 was given.

This was the end of this lecture.

8 Lecture 6, Wed, Feb. 10,2010

Talked about DFT (Fourier transform of an infinite discrete sequence also called DTFT) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$ where $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$. Notice that $X(e^{j\omega})$ is continuous in ω which has units radians. and is periodic of period 2π. Not every sequence $x[n]$ has DFT. It must be absolutely summable on its own for it to have DFT, i.e. $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$. This means $u(n)$, the unit step function do not have DFT.

If system is stable, then the $h(n)$ will have DFT (called frequency response)

Next, $H(e^{j\omega})$ was shown and its inverse was calculated. For a low pass filter. The result is sinc function.Note, low pass filter is over $-\pi, \pi$, and if the cut off frequency ω_c gets close to π, filter becomes all pass filter. We only need to look at region $-\pi, \pi$

Next talk about symmetry property of DTFT. Definition of even/odd for $x[n]$ and for $X(e^{j\omega})$

proofs given:

1. If $x[n]$ is real and even then $X\left(e^{j\omega}\right)$ is real and even.
2. If $x[n]$ is real and odd then $X\left(e^{j\omega}\right)$ is imaginary and odd
3. If $x[n]$ is imaginary and even then $X\left(e^{j\omega}\right)$ is imaginary and even
4. $x[n]$ is imaginary and odd then $X\left(e^{j\omega}\right)$ is real and odd

To remember these, note that when $x[n]$ is even, then $X\left(e^{j\omega}\right)$ follows $x[n]$, so not need to worry about this part. When $x[n]$ is odd, then $X\left(e^{j\omega}\right)$ is also odd, but it is opposite to what $x[n]$ is. When $x[n]$ real, $X\left(e^{j\omega}\right)$ becomes imaginary, and when $x[n]$ is imaginary, $X\left(e^{j\omega}\right)$ becomes real. So just remember the $x[n]$ odd part. Learn the proofs in notes.

Next talked about conjugate symmetric. sequence is CS, if $x[n] = x^*[-n]$ this means the real part of $x[n]$ is even and its imaginary part is odd. Next more properties about CS for $x[n]$ and $X\left(e^{j\omega}\right)$ are given. Not sure if these will come up in exam.

Learn how to find even and odd part of sequence.

Talked about sampling of $x(t)$ and sample and hold, and sampling function (impulse train)

This was the end of the lecture.

9 Lecture 7, Wed Feb. 17,2010

This lecture was all about sampling theory and Nyquist. Starting with an a periodic continuous time signal $x(t)$ and its Fourier transform, we sample $x(t)$ obtaining $x[n]$, a sequence of numbers, then find the DTFT of $x[n]$. This summarizes the relation between them.

time domain	frequency domain
$x(t)$ where t is time and in seconds $x(t)$ is aperiodic in t, and continuous $-\infty < t < \infty$	$X(\Omega)$ where Ω is radian frequency (rad/sec) $X(\Omega)$ is aperiodic in Ω, and continuous $-\infty < \Omega < \infty$
$x(t)$	$X_a(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t} d\Omega$
$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)\, e^{j\Omega t} d\Omega$	$X_a(\Omega)$

After sampling

sequence domain	frequency domain
$x[n]$ where n are integers $x[n]$ is aperiodic in n, and discrete $-\infty < n \in Integers < \infty$	$X(\omega)$ where ω is frequency (radians) $X(\omega)$ is periodic in ω, and continuous, period $= 2\pi$ and $-\infty < \omega < \infty$
$x[n]$	$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$
$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n} d\omega$	$X(\omega)$

Main things to notice: In time domain $t \to \Omega$, everything is continuous, and aperiodic. But once we sample, then the frequency domain becomes periodic. Notice that the unit of Ω is rad/sec and units of ω is radians only.

Next we derived the relation between $X(\Omega)$ and $X(\omega)$, the idea is this:

start with $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(\Omega)\, e^{j\Omega t} d\Omega$ and using the fact that at $t = nT$, where T is the sampling period, we replace t by nT in the above and write

$$x(nT) = x[n] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(\Omega)\, e^{j\Omega nT} d\Omega$$

But

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n} d\omega$$

So, now try to make the first expression above which involves $\int\limits_{-\infty}^{\infty}$ looks like the second integral $\int\limits_{-\pi}^{\pi}$.

This is done using 2 tricks. First, break the integral $\int\limits_{-\infty}^{\infty}$ into sums of integrals $\cdots+\int\limits_{-\frac{3\pi}{T}}^{-\frac{\pi}{T}}+\int\limits_{-\frac{\pi}{T}}^{\frac{\pi}{T}}+\int\limits_{\frac{\pi}{T}}^{\frac{3\pi}{T}}+\cdots$

This results in

$$x(nT)=x[n]=\frac{1}{2\pi}\sum_{r=-\infty}^{r=\infty}\int\limits_{\frac{(2r-1)}{T}\pi}^{\frac{(2r+1)}{T}\pi}X_a(\Omega)\,e^{j\Omega nT}d\Omega$$

Ok, this sounds cool, but we have not done anything yet.

Next, is the second more important trick, let $\Omega_1=\Omega-\frac{2\pi}{T}r$ (notice a minus sign here, in the lecture notes it was given as plus sign, but that would not work out as the final result was shown)

So now $d\Omega=d\Omega_1$, and when $\Omega=\frac{(2r-1)}{T}\pi$, then $\Omega_1=\frac{(2r-1)}{T}\pi-\frac{2\pi}{T}r=\frac{(2r-1)\pi-2\pi r}{T}=\frac{2r\pi-\pi-2\pi r}{T}=\frac{-\pi}{T}$ and when $\Omega=\frac{(2r+1)}{T}\pi$, then $\Omega_1=\frac{(2r+1)}{T}\pi-\frac{2\pi}{T}r=\frac{(2r+1)\pi-2\pi r}{T}=\frac{2r\pi+\pi-2\pi r}{T}=\frac{+\pi}{T}$, hence the above integral becomes

$$x(nT)=x[n]=\frac{1}{2\pi}\sum_{r=-\infty}^{r=\infty}\int\limits_{\frac{-\pi}{T}}^{\frac{+\pi}{T}}X_a\left(\Omega_1+\frac{2\pi}{T}r\right)e^{j\left(\Omega_1+\frac{2\pi}{T}r\right)nT}d\Omega_1$$

But Ω_1 is a dummy variable, so rename it. We might as well rename it back to Ω, so the above becomes

$$x(nT)=x[n]=\frac{1}{2\pi}\sum_{r=-\infty}^{r=\infty}\int\limits_{\frac{-\pi}{T}}^{\frac{+\pi}{T}}X_a\left(\Omega+\frac{2\pi}{T}r\right)e^{j\Omega nT}e^{j2\pi rn}d\Omega$$

But in $e^{j2\pi rn}$, we notice the exponent is always an integer (r is an integer, and so is n). Hence this is just 1, so the above becomes

$$x(nT)=x[n]=\frac{1}{2\pi}\sum_{r=-\infty}^{r=\infty}\int\limits_{\frac{-\pi}{T}}^{\frac{+\pi}{T}}X_a\left(\Omega+\frac{2\pi}{T}r\right)e^{j\Omega nT}d\Omega$$

But $\omega=\Omega T$ (the frequency axis scaling for the discrete case is a collapsed version of the frequency axis scaling of the continuous case. The scaling is determined by T). Hence, using the above, then the integral becomes

$$x(nT)=x[n]=\frac{1}{2\pi}\sum_{r=-\infty}^{r=\infty}\frac{1}{T}\int\limits_{-\pi}^{+\pi}X_a\left(\frac{\omega}{T}+\frac{2\pi}{T}r\right)e^{j\omega n}d\omega$$

Notice the $\frac{1}{T}$ coming out due to scaling effect. Now interchange the summation with the integration, we obtain

$$x(nT)=x[n]=\frac{1}{2\pi}\int\limits_{-\pi}^{+\pi}\left\{\frac{1}{T}\sum_{r=-\infty}^{r=\infty}X_a\left(\frac{\omega}{T}+\frac{2\pi}{T}r\right)\right\}e^{j\omega n}d\omega$$

Compare the above to the original expression for the inverse DTFT of $x[n]$ which is

$$x[n]=\frac{1}{2\pi}\int\limits_{-\pi}^{\pi}X(\omega)\,e^{j\omega n}d\omega$$

We see immediately that

$$X(\omega) = \frac{1}{T}\sum_{r=-\infty}^{r=\infty} X_a\left(\frac{\omega}{T} + \frac{2\pi}{T}r\right)$$

But $\frac{\omega}{T} = \Omega$, so the above becomes

$$X(\omega) = \frac{1}{T}\sum_{r=-\infty}^{r=\infty} X_a\left(\Omega + \frac{2\pi}{T}r\right)$$

Hence we see the final result, an important result, which is that the DTFT $X(\omega)$ of the samples can be obtained from the Fourier transform X_a of the signal itself from which the samples are taken using sampling rate T.

We just need to scale X_a and pick only X_a over the frequency Ω range of $-\pi \cdots \pi$ rad/sec, then divide the result by T to obtain $-\pi \cdots \pi$ radians. Then make copies of these by shifting them left and right by 2π at a time.

Hence, given X_a and T one can always generate $X(\omega)$ (need to write small program to show this).

10 Lecture 8, Monday Feb. 22,2010

2 main parts, the first was on properties of CTFT and DTFT. Handout was given. What happens to the transform when we do things to the time domain, such as scaling. Remember important thing, doing $x(2t)$ in time domain result is new signal which is squeezed version of $x(t)$, but we can still recover the original, but in discrete time effect of $x[2n]$ could be to produce a new sequence which we can't recover from the original. See notes for examples. So watch out for this., Learn the properties and proof of them. Learn how multiplication of summation and making it a double summation. Watch for indices.

Went over the dirac delta function $\delta(t)$ and how to use in. Sniffing property.

The rest of the lecture was on aliasing. A signal with 2 harmonics was given, we sample it under Nyquist, and using spectrum, the result is analyzed. Learn how to do this. I need to make a more detailed diagram as shown in the notes. learn the boundaries of the spectrum and the window used.

I made some notes on this, see below in the study notes section.

HW1 solutions returned.

11 Lecture 9, Wednesday Feb. 24, 2010

This lecture was all about Z transform and region of convergence ROC. We looked at left sides, right sides, and 2 sides, and how to find the ROC for each. Nice trick to help us remember the ROC was shown, which is: For right-sided sequence, look to the right, so the ROC is outside some circle. And for left-sided sequence, look to the left, hence the ROC is inside some circle.

For 2 sided sequence, the ROC will be inside the space of 2 circles, one larger than the other. It is also possible that the ROC can not exist for 2 sided.

Notes are given, We went over residue theorem, but it wont be on exam.

Important properties of Z transform given, but we did not go over some of them, they are in the notes.

Exam will be in 2 weeks time.

12 Lecture 10, Monday March 1, 2010

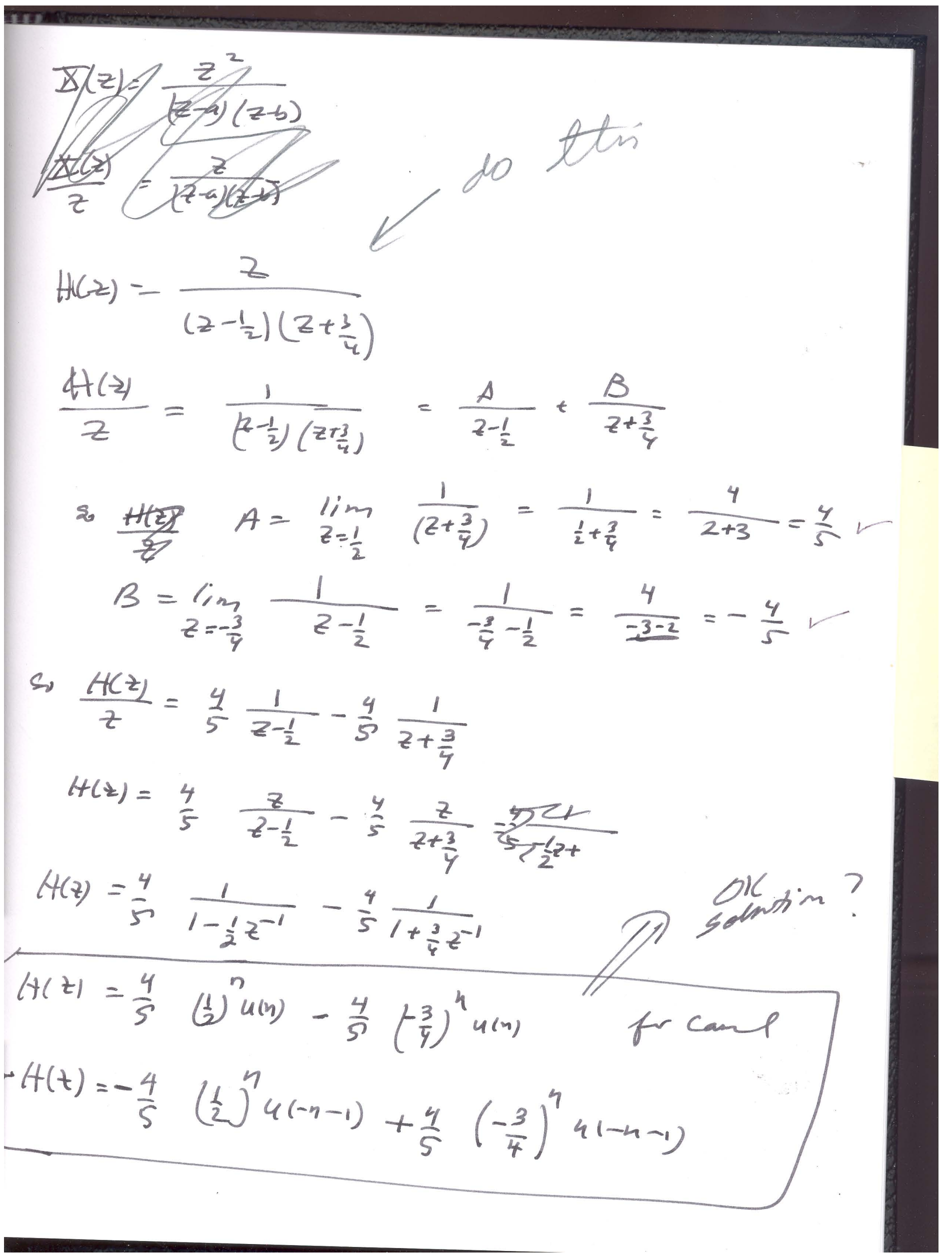
This lecture was on finding the inverse Z transform using contour integration with the trick of using $z \rightarrow \frac{1}{p}$ for left sided signals to make it easier to do. We send most of the time looking at this. This is a little confusing lecture for me, and need more studying, but it looks like this will not be on exams. Most likely if we have to find inverse Z transform, then use partial fractions or long division. So for now, I will not spend to much time on this unless I think it will be on the exam.

Solution to HW 2 was given.

13 Lecture 11, Wednesday March 3, 2010

On partial fractions. Remember this: Always start by multiplying both sides by z^{-1}, so we end up with

$\frac{X(z)}{z}$, and do partial fraction on the RHS, which now should be with one less zero in the numerator. Once done, then find $X(z)$ by multiplying the result with z again. Here is an example:



Looked at case with multipoles for partial fractions. This requires derivatives. See the notes. I do not think we will get one like this in the example.

Then went over HW3, the sampling problem. This is important. Remember that aliasing happens at frequencies which are $f_0 + kf_s$ where f_0 is the frequency of the signal (highest) and f_s is the sampling frequency, and $k = 1, 2, 3, etc.$.

Looked at more partial fractions. Just multiply both sides by z^{-1} and should be ok.

14 Lecture 12, Monday March 8, 2010

Went back to residue integration a little. Then covered initial value theorem. This is useful to find $x(0)$ given $X(z)$, where $x(0) = \lim_{z\to\infty} X(z)$, to find $x(1)$, the trick is to find $x(0)$ first, then if it is zero, then multiply $X(z)$ by z, this shifts everything back by 1, then apply the theory again to $x(0)$, which is now really $x(1)$.

Then went over a quick way to plot $|H(\omega)|$ from the location of poles and zeros around the unit circle. Imagine moving around the circle from 0 to π, then consider a pole to be where $|H(\omega)|$ is very large, and a zero is where $|H(\omega)|$ is very small, then one can draw a rough sketch of the $|H(\omega)|$

End of lecture.

15 Lecture 13, Wed March 10, 2010

First midterm (a little hard and long)

16 Lecture 14, Monday March 15, 2010

Went over solution of midterm. For next midterm, good chance we will have a question on periodic convolution and z-transforms (no question on z-transform in the first midterm). Average of class was 70. I need to buy myself one of those fancy calculators and learn how to use them, so I can use it to verify my solution in the exam.

17 Lecture 15, Wed March 17, 2010

HW4 key solution handed in.

Handout D handed in (4th handout). This lecture was mainly about 2 things: Periodic convolution, and DFT.

If we have a sequence of numbers, we can find the DFT (Discrete Fourier Transform) of this sequence. We assume the sequence of numbers repeat (just for convenience). Write little delta over x to indicate periodic. These are the definitions to know

$$\tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k)e^{j\left(\frac{2\pi}{N}\right)nk} \qquad n = 0\cdots N-1$$
$$\tilde{X}(k) = \sum_{n=0}^{N-1}\tilde{x}(n)\,e^{-j\left(\frac{2\pi}{N}\right)nk} \qquad k = 0\cdots N-1$$

N is the length of the sequence $\tilde{x}(n)$ and $\tilde{X}(k)$ is the DFT of $\tilde{x}(n)$.

One important thing to know: If we find the Z transform of $\tilde{x}(n)$, then sampling the Z transform around the unit circle, we get the values of each $\tilde{X}(k)$.

How to sample the Z transform? start from zero angle and move anticlockwise for an angle θ where $\theta = \frac{2\pi}{N}$ and read the Z transform at that polar coordinate. This gives values of $\tilde{X}(k)$. So $\tilde{X}(0)$ is Z transform of $\tilde{x}(n)$ at angle zero, and $\tilde{X}(1)$ is Z transform of $\tilde{x}(n)$ at angle $\frac{2\pi}{N}$ and $\tilde{X}(2)$ is Z transform at the angle $2\frac{2\pi}{N}$ and so on

Then went over properties of $\tilde{x}(n)$, but we are really more interested in properties of $\tilde{X}(k)$.

Went over example of periodic convolution (might be on second midterm).

It is easier than linear convolution, since both sequences will always start at $n = 0$ and have the same period. So, just flip one, and remember to only look at one period.

FFT is just an implementation to determine DFT.

Most use of FFT is for doing fast convolution. We studies FFT is details in EE 518 that I took last year.

This was the end of this lecture.

18 Lecture 16, Monday March 22, 2010

Continue studying DFT.

Important things to know: We implement linear convolution by doing circular convolution. Assume we have 2 sequences x_1 and x_2, both of same length, say N. We can use circular convolution to implement linear convolution. but need $2N-1$ as the length to do it using circular.

But why do we do this? because circular convolution can then be found in a fast this way: Multiply the DTF of the 2 sequences, then find the inverse DFT.

This is the circular convolution of the 2 sequences. Which is also the linear convolution (if we have the $2N-1$ length sorted out first). We can always append zeros to the end of the sequences to make the length be $2N-1$.

Since we have fast algorithm to do DFT and inverse DFT (example, FFT), then this is a way to quickly find linear convolution of 2 sequences.

We had examples showing how to do circular convolution. Make sure to practice this more as it will be on exam.

HW 6 assigned today. We are now going over Handout E on DFT.

19 Lecture 17, Wed March 24, 2010

Continue on handout E, page 11. We can do linear convolution using circular convolution. Talked about realization of a system. Given the difference equation of a discrete system, how to connect delay elements, adders, subtract to implement this difference equation. Finished handout E.

Handout F given, on flow graphs for digital system.

spring break !

20 Lecture 18, Monday April 6,2010

On page 6, handout F. Realization forms of filters. cascade forms of order 2, direct form 1, and 2. Learn how to given H(z), to generate direct forms.

Talked about linear phase FIR

21 Lecture 19, Thursday april 8,2010

midterm 2

22 Lecture 20, Monday april 12,2010

Went over exam problem

23 Lecture 21, Wednesday april 14,2010

More on direct forms, frequency sampling structures, mapping from H(s) to H(z)

24 Lecture 22, Monday april 19,2010

More on mapping from H(s) to H(z), impulse invariance and bilinear transformation. Frequency wrapping

25 Lecture 23, Wednesday april 21,2010

Started on digital filters design, impulse invariance, specifications

26 Lecture 24, Monday april 26,2010

Continue with digital filter design, go over example, handout H

27 Lecture 25, Wednesday april 28,2010

Movie illustrations from a DSP book

28 Lecture 26, Monday may 3,2010

Went over geometric approach to filter design

29 Lecture 27, Wednesday may 5,2010

More on geometric approach to filter design, find notch filter specs from geometry.

Started on FIR filter design, but will not be on exam. If we need linear phase, must use FIR. FIR used in 2D application (digital images), used windows for design. Two effects to watch for: smearing (from main loop), and oscillation (from side loops).

30 Lecture 28, Monday may 10,2010

More intro on FIR, windows used, frequency sampling design. Some review of FIR.

End of semester.