Handout I

4/27/10

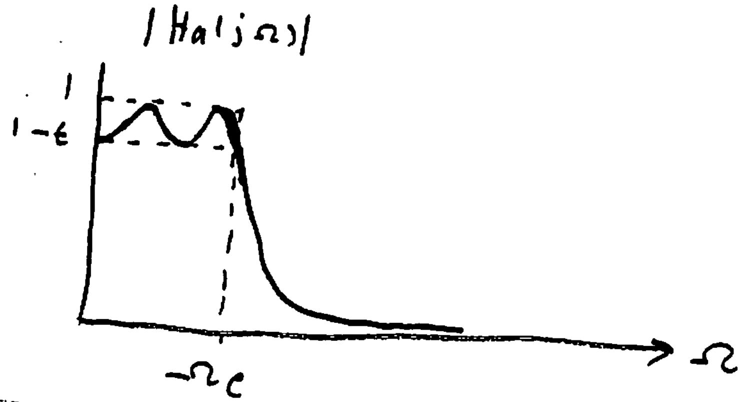
Not on Exam.

Digital Chebyshev Filter

Magnitude of the freq. resp. is either equiripple in the passband and monotonic in the stopband.

OR monotonic " " " " equiripple " " ".

Usually ⇒ lower-order filter



$$\boxed{|H_a(\Omega)|^2 = \frac{1}{1+\epsilon^2 V_N^2(\Omega/\Omega_c)}}$$

continuous filter defined in terms of square magnitude.

$V_N(x)$: N^{th} order Chebyshev polynomial

$$V_N(x) = \cos(N \cos^{-1} x)$$

$$V_0(x) = 1$$

$$V_1(x) = \cos(\cos^{-1} x) = x$$

$$V_2(x) = \cos(2 \cos^{-1} x) = 2x^2 - 1$$

$$V_{N+1}(x) = 2x V_N(x) - V_{N-1}(x)$$

$0 \leqslant x \leqslant 1 \quad \Rightarrow \quad 0 \leqslant V_N^2(x) \leqslant 1$

$\begin{cases} x > 1 \Rightarrow \cos^{-1} x \text{ becomes imaginary} \\ V_N(x) \text{ behaves as a hyperbolic cosine} \end{cases}$

$\therefore \begin{cases} 0 \leqslant \Omega/\Omega_c \leqslant 1 \Rightarrow |H_a(\Omega)|^2 \text{ ripples between } 1 \text{ and } \frac{1}{1+\epsilon^2} \\ \Omega/\Omega_c > 1 \Rightarrow |H_a(\Omega)|^2 \text{ decreases monotonically} \end{cases}$

Design specs $\begin{cases} \epsilon : \text{ allowable passband ripple} \\ \Omega_c : \text{ specified by the desired cutoff freq.} \\ N : \text{ chosen to meet stopband specifications} \end{cases}$

Poles of the Chebyshev filter

lie ~~...~~ on an ellipse in the s-plane which is defined by two circles corresponding to the minor and major axes of the ellipse.

$\begin{cases} \text{Radius of minor axis} = a\,\Omega_c \\ \text{" \quad " major "} = b\,\Omega_c \end{cases}$

$\begin{cases} a = \frac{1}{2}\left(\alpha^{1/N} - \alpha^{-1/N}\right) \\ b = \frac{1}{2}\left(\alpha^{1/N} + \alpha^{-1/N}\right) \end{cases}$

$\alpha = \epsilon^{-1} + \sqrt{1 + \epsilon^{-2}}$

Bilinear design

Assuming $T = 1$ $\qquad \Omega = \frac{2}{T} \tan\left(\frac{\omega}{2}\right)$

$$\Rightarrow \begin{cases} 20 \log \left| H_a\left(j\, 2 \tan\left(\frac{.2\pi}{2}\right)\right)\right| \geq -1 & \text{I} \\ 20 \log \left| H_a\left(j\, 2 \tan\left(\frac{.3\pi}{2}\right)\right)\right| \leq -15 & \text{II} \end{cases}$$

$\boxed{\Omega_c = 2 \tan(.2\pi/2)}$ $\qquad$ I $\Rightarrow$ $\qquad |H_a(j\Omega_c)|^2 = \frac{1}{1+\epsilon^2} = 10^{-.1}$

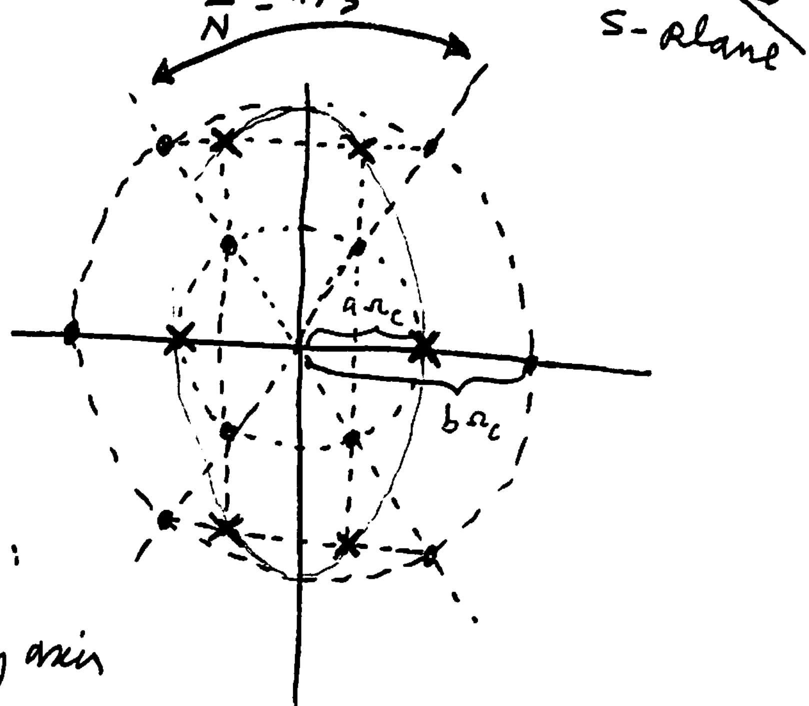
$\Rightarrow \boxed{\epsilon = 0.50885}$

II $\Rightarrow N = 4$

$$H_a(s) = \frac{.04381}{(s^2 + .1814s + .4166)(s^2 + .4378s + .1180)}$$

$$H(z) = H_a(s)\Big|_{s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = \frac{---}{---} \quad \text{here } T = 1$$

See pp. 223 - 224 for $H(z)$ & the plots



Ex. $N = 3$

Poles ?

Find points separated by π/N in angle on the two circles, and symmetrical with respect to the imaginary axis

properties:

1. No points on the imaginary axis
2. a point on real axis only for N odd

(So far: similar to Butterworth filter)

Poles { ordinate specified by the points on the major circle
abscissa " " " " " minor "

Design $\xrightarrow{\text{same as before}}$ $\begin{cases} 20\log_{10}|H(e^{j.2\pi})| \geq -1 \\ 20\log_{10}|H(e^{j.3\pi})| \leq -15 \end{cases}$

Impulse Invariant

$\Rightarrow \begin{cases} 20\log|H_a(j.2\pi)| \geq -1 \\ 20\log|H_a(j.3\pi)| \leq -15 \end{cases}$

Assuming $T=1 \Rightarrow T^{-1}\Omega = \omega$

meet spec.'s at $.2\pi$

equiripple for $0 \leq \Omega \leq .2\pi$

$$|H_a(\Omega)|^2 = \frac{1}{1+\epsilon^2\left[\cos(N\cos^{-1}\frac{\Omega}{\Omega_c})\right]^2}$$

At $\boxed{\Omega = \Omega_c = .2\pi}$ $\quad |H_a(j.2\pi)|^2 = \dfrac{1}{1+\epsilon^2} = 10^{-2(1/20)} = 10^{-.1}$

$\Rightarrow \epsilon^2 = \dfrac{1}{10^{-.1}} - 1 = 10^{.1} - 1 = .2589$

$\Rightarrow \boxed{\epsilon = 0.50885}$

~~Passband~~ Stopband $20 \log |H_a(j.3\pi)| \leq -15$

$$|H_a(j0.3\pi)|^2 \leqslant 10^{-1.5} \cong 0.031623$$

$$|H_a(j0.3\pi)|^2 = \left| \frac{1}{1 + .2589 \left[\cosh(N \cosh^{-1}(1.5))\right]^2} \right.$$

Ω/Ω_c

$N=3 \Rightarrow |H_a(j0.3\pi)|^2 \cong .0455$ ~ -13.42 db

$N=4 \Rightarrow |H_a(j.3\pi)|^2 \cong 0.00694$ ~ -21.58 db

$\therefore$ Take $\boxed{N = 4}$

$$\alpha = \epsilon^{-1} + \sqrt{1+\epsilon^{-2}} = 4.1702$$

$$a = \frac{1}{2}\left(\alpha^{1/4} - \alpha^{-1/4}\right) = .3646$$

$$b = \frac{1}{2}\left(\alpha^{1/4} + \alpha^{-1/4}\right) = 1.0644$$

Find the poles.

Take ~~all~~ left-half-plane poles to form $H_a(s)$

$$H_a(s) = \frac{0.038286}{(s^2 + 0.4233s + 0.1103)(s^2 + 0.1753s + 0.3894)}$$

chosen to give the correct ~~DC gain~~ gain at $\Omega = \Omega_c$

$$= \sum_{k=1}^{4} \frac{A_k}{s - s_k}$$

$$H(z) = \sum_{k=1}^{4} \frac{TA_k}{1 - e^{s_k T} z^{-1}} = \cdots$$

see ~~page~~ PP. 221 - 222 for $H(z)$ & plots