3.2 a)

1) $\mathcal{F}[\tilde{x}(n+m)] = \sum_{n=0}^{N-1} \tilde{x}(n+m) W_N^{kn} = \sum_{n'=m}^{N-1+m} \tilde{x}(n') W_N^{k(n'-m)}$

$= W_N^{-km} \tilde{X}(k)$

2) $\mathcal{F}[\tilde{x}^*(n)] = \sum_{n=0}^{N-1} \tilde{x}^*(n) W_N^{kn} = \left(\sum_{n=0}^{N-1} \tilde{x}(n) W_n^{-kn}\right)^* = \tilde{X}^*(-k)$

3) $\mathcal{F}[\tilde{x}^*(-n)] = \sum_{n=0}^{N-1} \tilde{x}^*(-n) W_N^{kn} = \left(\sum_{n'=0}^{-N+1} \tilde{x}(n') W_N^{kn'}\right)^* = \tilde{X}^*(k)$

4) $\mathcal{F}[Re[\tilde{x}(n)]] = \sum_{n=0}^{N-1} \frac{\tilde{x}(n)+\tilde{x}^*(n)}{2} W_N^{kn} = \frac{1}{2}[\tilde{X}(k)+\tilde{X}^*(-k)] = \tilde{X}_e(k)$

5) $\mathcal{F}[j\,Im[\tilde{x}(n)]] = \sum_{n=0}^{N-1} \frac{\tilde{x}(n-\tilde{x}^*(n)}{2} W_N^{kn} = \frac{1}{2}[\tilde{X}(k)-\tilde{X}^*(-k)] = \tilde{X}_0(k)$

b)

1.) $Re[\tilde{X}(k)] = \frac{\tilde{X}(k)+\tilde{X}^*(k)}{2} \underset{\uparrow}{=} \frac{\tilde{X}^*(-k)+\tilde{X}(-k)}{2} = Re[\tilde{X}(-k)]$

for $\tilde{x}(n)$ real $\tilde{X}(k) = \tilde{X}^*(-k)$ & $\tilde{X}^*(k) = \tilde{X}(-k)$ from 2 & 3 above

2) $Im[\tilde{X}(k)] = \frac{\tilde{X}(k)-\tilde{X}^*(k)}{2j} = \frac{\tilde{X}^*(-k)-\tilde{X}(-k)}{2j} = -Im[\tilde{X}(-k)]$

3) $|\tilde{X}(k)| = (\tilde{X}(k)\tilde{X}^*(k))^{1/2} = (\tilde{X}^*(-k)\tilde{X}(-k))^{1/2} = |\tilde{X}(-k)|$

4) $Arg\,\tilde{X}(k) = \tan^{-1}\frac{Im\,\tilde{X}(k)}{Re\,\tilde{X}(k)} = \tan^{-1}\left(-\frac{Im\,\tilde{X}(-k)}{Re\,\tilde{X}(-k)}\right) = -Arg\,\tilde{X}(-k)$

standard.

3-7 DFT of N-seq.'s

a) $x(n) = \delta(n)$

$$X(k) = \sum_{n=0}^{N-1} \delta(n) W_N^{kn} = \begin{cases} 1 & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

Know these!

b) $x(n) = \delta(n-n_0) \qquad 0 < n_0 < N$

$$X(k) = \sum_{n=0}^{N-1} \delta(n-n_0) W_N^{kn} = \sum_{\ell=-n_0}^{N-1-n_0} \delta(\ell) W_N^{k(\ell+n_0)}$$

$$= \begin{cases} W_N^{kn_0} & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

c) $x(n) = a^n \qquad 0 \le n \le N-1$.

$$X(k) = \sum_{n=0}^{N-1} a^n W_N^{kn} = \frac{1-(aW_N^k)^N}{1-aW_N^k} = \begin{cases} \dfrac{1-a^N}{1-ae^{-j\frac{2\pi k}{N}}} & 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

standard.

3-10 10 KHz 1024 samples

$$\Delta\Omega = \frac{\Delta\omega}{T} = \frac{2\pi}{NT} = \frac{2\pi f}{N} = \frac{2\pi \times 10^4}{1024} \text{ rad/s}$$

$$\therefore \Delta f = \frac{10^4}{1024} \sim 9.7 \text{ Hz}.$$

One revolution around the unit ~~sample~~ circle is equivalent to 10 KHz. Using a 1024 point DFT.

$$\therefore \Delta f = \frac{10^4}{1024} \sim 9.7 \text{ Hz}$$

1 around the circle $\equiv$ 9.7 Hz.

3-14 a) $x(n) = -x(N-1-n)$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{-kn}$$

$$X(0) = \sum_{n=0}^{N-1} x(n) = \frac{1}{2}\sum_{n=0}^{N-1}\left[x(n) - x(N-1-n)\right]$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n) - \frac{1}{2}\sum_{n'=N-1}^{0} x(n') = \frac{1}{2}\sum_{n=0}\left[x(n) - x(n)\right] = 0$$

b) $\left.\begin{array}{l} N \text{ : even} \\ x(n) = x(N-1-n) \end{array}\right\} \Rightarrow X\left(\frac{N}{2}\right) = 0$

$$X\left(\tfrac{N}{2}\right) = \sum_{n=0}^{N-1} x(n)\, W_N^{\frac{N}{2}n} = \sum_{n=0}^{N-1} x(n)\, e^{-j\pi n} = \sum_{n=0}^{N-1} x(n)(-1)^n$$

$$= \frac{1}{2}\sum_{n=0}^{N-1}\left[x(n) + x(N-1-n)\right](-1)^n$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n)(-1)^n + \frac{1}{2}\sum_{n'=0}^{N-1} x(n')(-1)^{N-1-n'}$$

$$= \frac{1}{2}\sum_{n=0}^{N-1} x(n)(-1)^n + \frac{1}{2}(-1)\sum_{n'=0}^{N-1} x(n')(-1)^{n'} = 0$$

3-15

$$\mathcal{F}_D\left[X(k)\right] = \sum_{k=0}^{N-1} X(k)\, W_N^{kn}$$

$$= \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} x(m)\, W_N^{(n+m)k}$$

$$= \sum_{m=0}^{N-1} x(m) \underbrace{\sum_{k=0}^{N-1} W_N^{(n+m)k}}_{=\begin{cases} N & \text{if } m=-n \\ 0 & \text{otherwise}\end{cases}} = N\big((x(-n))\big)_N\, R_N(n)$$

3-16

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

$$\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} x(n) x^*(n) = \sum_{n=0}^{N-1} x(n) \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) W_N^{kn}$$

using 3.26

$$= \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) \sum_{n=0}^{N-1} x(n) W_N^{kn} = \frac{1}{N} \sum_{k=0}^{N-1} X^*(k) X(k)$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$