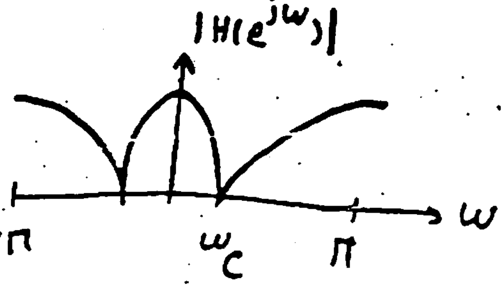
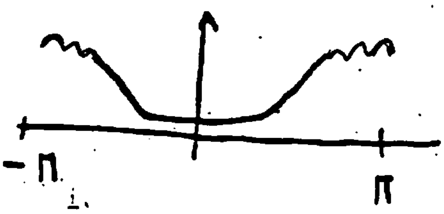
EGEE 420 H.W. # 9 Sol.

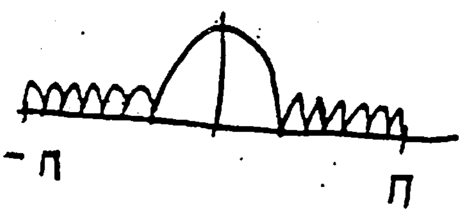
①

(a) 

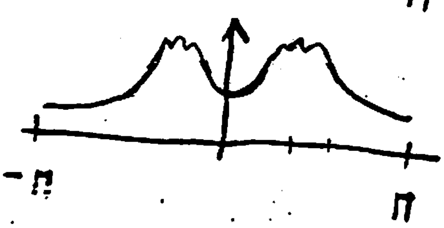
Notch filter

(b) 

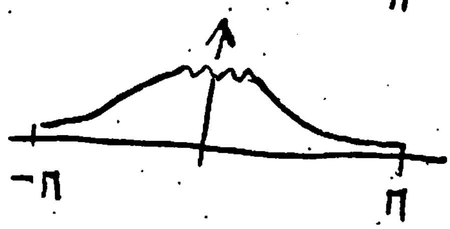
High-pass filter

(c) 

Comb filter
LP filter

(d) 

Band-pass filter

(e) 

Low-pass filter

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② $$\boxed{H_a(s) = \frac{s+2}{(s+2)^2+4}} = \frac{1/2}{s+2+2j} + \frac{1/2}{s+2-2j}$$

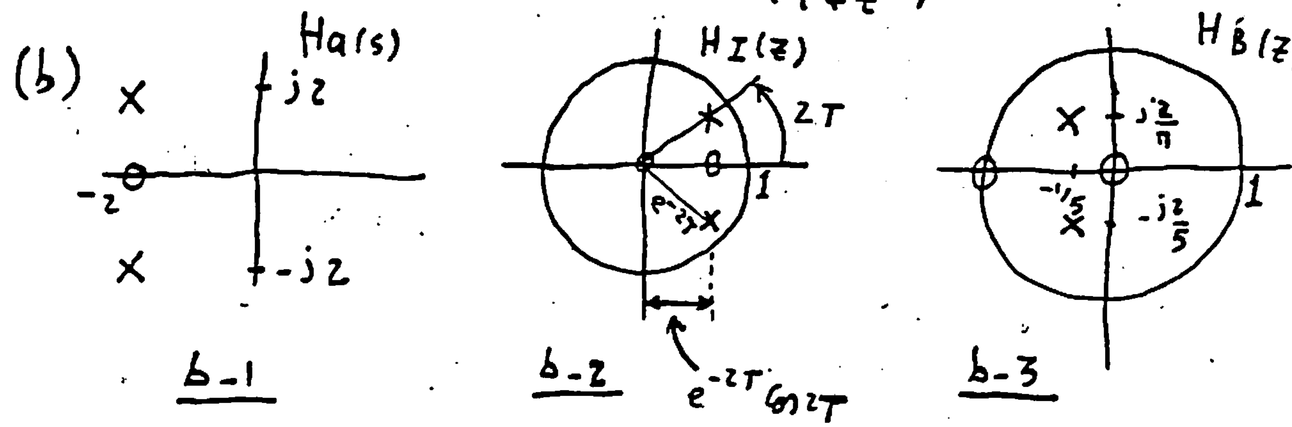
<u>a-1</u> $$H_I(z) = \frac{(1/2)\,T}{1-e^{(-2-2j)T}z^{-1}} + \frac{(1/2)\,T}{1-e^{(-2+2j)T}z^{-1}}$$

$$= \frac{z\,(z - e^{-2T}\cos 2T)\,T}{(z - e^{-2T}e^{-2jT})(z - e^{-2T}e^{2jT})}$$

$$H_B(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$, For simplicity take $T=1$

$$= \frac{s+2}{(s+2)^2+4}\Bigg|_{s=2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{z(z+1)}{\left(z+\frac{1}{5}+j\frac{2}{5}\right)\left(z+\frac{1}{5}-j\frac{2}{5}\right)}$$

(b)



b-1 b-2 b-3

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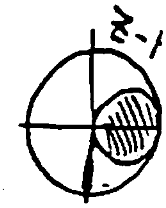
(3) $s = \dfrac{1-z^{-1}}{T} \Rightarrow z = \dfrac{1}{1-sT}$, At $s=j\Omega$ $z = \dfrac{1}{1-j\Omega T}$

or $z = \frac{1}{2}\left[1 + \dfrac{1+j\Omega T}{1-j\Omega T}\right] = \frac{1}{2}\left[1 + e^{j2\tan^{-1}(\Omega T)}\right]$

a circle at $z=1/2$ & radius $=1/2$



∴ The left-half-plane (including $j\Omega$ axis) maps into the unit circle. Thus stable $H_a(s) \longrightarrow$ stable $H(z)$

Alternatively $z = \dfrac{1}{1-sT}$, $s = \sigma + j\Omega$, $z = \dfrac{1}{1-\sigma T - j\Omega T}$

$|z|^2 = \dfrac{1}{(1-\sigma T)^2 + \Omega^2 T^2}$, so for $\sigma < 0$, $|z| < 1$. Thus stable $H_a(s) \longrightarrow$ Stable $H(z)$.

Stable $H(z)$ does not always result in an stable $H_a(s)$.
Counter example , Let $H(z)$ be stable for $|z| > 1$,
say $z = 2$, $z^{-1} = 1/2 \Rightarrow s = \dfrac{1-z^{-1}}{T} > 0$
or let $R_e[s_k] = \frac{3}{T} > 0 \Rightarrow |z_k| < 1$ ∴ $|z_k| < 1 \not\Rightarrow Re[s_k] < 0$